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ELEMENTARY VORTEX: ITS DYNAMICAL ROLES IN TURBULENCE

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ABSTRACT: The elementary vortex is the smallest vortical structure commonly observed in various kinds of turbulence. It plays important roles in turbulence dynamics, such as mixing and vortex generation. A series of studies on the elementary vortex is reviewed, including an eduction method of the vortex by the low-pressure criterion and enhancement of stretching of fluid lines.

1. INTRODUCTION

Turbulence is composed of many vortical motions of different sizes and shapes. The knowledge of the physical characteristics of individual vortices is helpful for understanding of turbulence dynamics. It is generally recognized that tubular swirling regions of high vorticity exist in various kinds of turbulence. They are nicknamed worms in isotropic turbulence, streamwise vortices in wall turbulence, rolls and ribs in free-shared turbulence, and so on. Although the shape of the cross-section and the length of such tubular vortices vary from turbulence to turbulence, the size of the cross-section and the swirl velocity obey, in average, the Kolmogorov scaling law. They are regarded as the smallest vortical structure in turbulence, so that they are called the elementary vortex. The elementary vortex plays significant roles in turbulence dynamics, such as generation and sustenance of turbulence, diffusion and mixing, etc. Here, we review a series of studies on the elementary vortex performed these years by our research group.[1−5]

An eduction method of the elementary vortex from a turbulent field is introduced in §2. A vortex is identified as a low-pressure region and the physical characteristics are discussed. The dynamical roles of the elementary vortex in deformation and stretching of fluid lines are examined in §3. Finally, a future direction in the study of the elementary vortex is viewed in §4.

2. LOW-PRESSURE CRITERION

Among a variety of vortices we focus here the tubular vortices. The pressure is lowered at the central axes of tubular vortices to counter-balance the centrifugal force caused by the swirling motion around the vortices. This fact motivates us to identify and extract the tubular vortices as the low-pressure regions, which are found numerically by searching lines of local minimum (in a plane perpendicular to the line) of pressure. First, we define a plane, at any point $\boldsymbol{x}$ in a flow, that is perpendicular to the eigen-vector of the pressure hessian having the smallest eigen-value. Then, we check whether the pressure on this plane takes a minimum at $\boldsymbol{x}$ or not. The axis of a vortex is constructed by connecting properly the minimum-pressure points thus obtained. The pressure on the plane is concave in the vicinity of $\boldsymbol{x}$. Generally, there is an inflection closed line surrounding $\boldsymbol{x}$ on this plane, across which the concavity of the pressure changes the sign. In the three-dimensional space it gives an inflection surface which defines the core boundary of the elementary vortex (Fig. 1). See references [1,2] for the numerical algorithm.

This low-pressure criterion is applied to an isotropic turbulence, and the axes of elementary vortices are depicted in Fig. 2. Spirals attached to some of the axes are the streamlines which are viewed in frames moving with velocity of the nearest part of the axes. Note that the spiralling streamlines are generally not observed in a fixed frame.

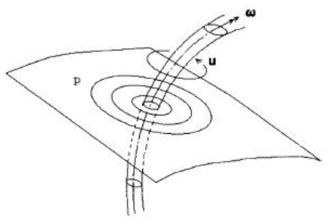


Fig. 1: An elementary vortex. The axis and the core boundary are defined by a line of local minimum of pressure and a surrounding inflection surface.

Visualization of the elementary vortex by the low-pressure criterion reveals some new interesting features of structure and dynamics of vortices.

First, every vortex accompanies the double vortex layers wrapping it. In Fig. 3 we show the vorticity magnitude in a cross-section of a vortex located at the center. High-vorticity is observed not only in the central circular region but also in two surrounding round layers. The vorticity in the central region is parallel to the vortex axis, whereas that in the layers has large angles with the axis. These layers play important roles in enhancement of mixing and of energy dissipation.

Secondly, the physical characteristics of the elementary vortex are evaluated quantitatively. Their diameter in cross-section is about ten times the Kolmogorov length η, and the swirl velocity is about three times the Kolmogorov velocity.

Thirdly, two vortices often approach each other in an anti-parallel manner, i.e. their vorticities are opposite. Therefore, they tend to be broken down due to cross diffusion of vorticity. Anti-parallel pairs of vortices advect quite rapidly in a turbulent flow with mutually induced velocity, which play significant roles in stretching of fluid lines (see §3).

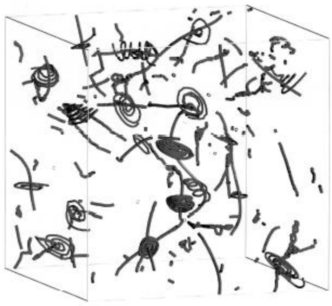


Fig. 2: Axes of elementary vortices and streamlines relative to the axes in a freely decaying turbulence.[1]

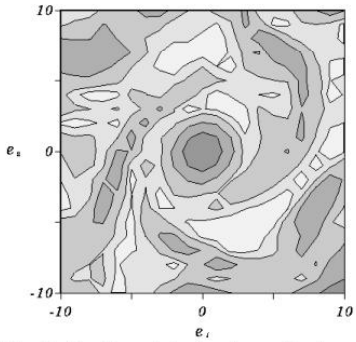
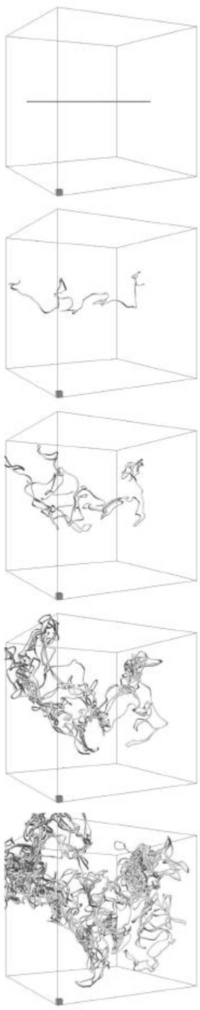


Fig. 3: Double spiral structure. Contours of vorticity magnitude on a cross-section of an elementary vortex which is located at the center. Darker shade implies larger values. The coordinate of the cross-section is measured in the unit of the grid width Δx taken in the numerical simulation. The Kolmogorov length is $2\Delta x$.

3. STRETCHING OF FLUID LINES



The transport, such as mixing and diffusion, of material objects is one of the most important dynamical properties of fluid flows. The reliable prediction and efficient control of transport coefficients are desirable in various industrial processes as well as in conservation of environment. The efficiency of transport varies depending on the state of fluid motions. It is much enhanced in turbulent states in comparison with laminar states. The complexity of fluid motions, especially, of turbulence, however, makes it very difficult to reveal the mechanism of mixing though some statistically averaged properties have been established. Here, we investigate the deformation of a fluid line, as a fundamental material object, in turbulence with special attention to the dynamical role of the elementary vortex.[6−8]

A fluid line in a statistically stationary turbulence, initially straight, is traced numerically. A temporal evolution of the fluid line is drawn in Fig. 4. Time elapses from top to bottom. Large cubes represent the whole computational domain, the side length of which is about $2\mathcal{L}$, where $\mathcal{L}$ is a length which characterizes energy-containing-scale motions. The side length of small cube is 10η which is comparable with the mean diameter of the elementary vortex. As time progresses, the line gets more and more convoluted. The total length increases, in average, exponentially in time as $L(t) \propto \exp[\gamma t/\tau_\eta]$, where $\gamma = 0.17$ and τ_η is the Kolmogorov time. The radius of curvature is comparable with the mean diameter of the elementary vortex, suggesting that the elementary vortex is involved in deformation of fluid lines.

Anti-parallel pairs of elementary vortices may give a significant contribution to the stretching. In Figs. 5 we show two different views of three nearly-aligned vortices and deformed fluid lines around them.[8] In the top view (Fig. 5(a)) observed are the characteristics of stretching on cross-sections of the vortex cluster, where the central vortex induces a clockwise swirling flow, and the other two vortices do anti-clockwise ones. Two clusters, i.e. a left vertical and the central horizontal white lines, of strongly stretched parts of a fluid line form a T-shape. In a side view (Fig. 5(b)) almost all the strongly stretched parts of the fluid line are parallel to each other and perpendicular to the vortices. In other words, the strong stretching can be regarded as a two-dimensional phenomenon.

Fig. 4. Temporal evolution of a fluid line. Time elapses from top to bottom by $0.2\mathcal{T}$, where $\mathcal{T}$ is an energy-containing-scale time. $R_\lambda = 83$.

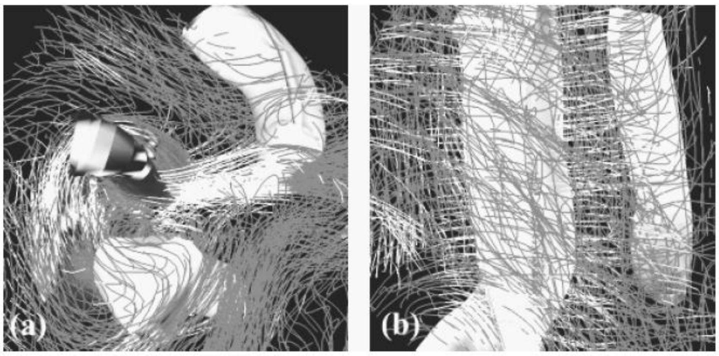


Fig. 5: Stretching enhancement by a vortex cluster. Three thick tubes represent the elementary vortices identified by the low-pressure criterion. Many lines represent a single fluid line with white for strongly stretched parts. Three vortices nearly aligned are seen from two different angles. (a) Top view. The central vortex rotate clockwise and the other two counter-clockwise. (b) Side view. Vorticity of the central vortex points downward and the other two upward.

4. PROSPECTIVE

The elementary vortex educed by the low-pressure criterion is identified individually. Any physical structure can be analyzed by use of various visualization techniques. We have developed an algorithm to trace the temporal evolution of each vortex.[4] A numerical simulation/visualization analysis using the elementary vortex is expected to be useful for understanding of the mechanism of generation, development and decay of vortical structures in turbulence.

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