Proceedings of the 12th Asian Congress of Fluid Mechanics ACFM No.1000***
18-21 August 2008, Daejeon, Korea

SMALL-SCALE STATISTICS IN HIGH REYNOLDS NUMBER TURBULENCE – A STUDY BY DIRECT NUMERICAL SIMULATION –

Y. Kaneda

Department of Computational Science and Engineering, Nagoya University, Japan, kaneda@cse.nagoya-u.ac.jp

ABSTRACT: A review is given on the study of small-scale statistics in high Reynolds number turbulence by using the data of high resolution direct numerical simulation of incompressible turbulence with the number of grid points and the Taylor-scale Reynolds number up to 4096^3 and approximately 1130, respectively. An emphasis is put on the dependence of the small-scale statistics on the Reynolds number and the scale. A review is also given on a theory of the anisotropy in small-scale statistics in turbulent shear flow, stably stratified turbulence and magneto-hydrodynamic turbulence under a strong uniform magnetic field.

1. INTRODUCTION

In 1941, Kolmogorov[9] (hereafter referred as K41) proposed an idea of universal equilibrium range and universality in small-scale statistics in turbulence at high Reynolds number far from flow boundaries. According to this idea, there is a certain kind of universality in the statistics, which is insensitive to the details of the large-scale flow conditions. The understanding of the universality may provide us with a sound basis for constructing turbulence models. The idea has been supported by experiments and direct numerical simulations (DNSs), and it is at the heart of modern theories of turbulence. The idea also justifies the use of simple boundary conditions, such as periodic boundary conditions, and simple large-scale forcing for the study of the small-scale statistics in high Reynolds number turbulence by DNS.

This paper presents a review of the analysis of high resolution DNS data of incompressible turbulence, with an emphasis on the dependence of the small-scale statistics on the Reynolds number *Re* and the scale. The DNSs consist of two series of simulations of forced turbulence obeying the Navier-Stokes equation in a periodic box with the number of the grid points up to 4096^3; one is with $k_{\max}\eta \sim 1$ (Series 1) and the other is with $k_{\max}\eta \sim 2$ (Series 2), where $k_{\max}$ is the highest wave number in each simulation, and η is the Kolmogorov length scale.[5,12] The DNSs were performed on the Earth Simulator with sustained performance up to 16.4 Tflops, and are based on a spectral method free from alias error. In the 4096^3 DNSs, the Taylor-scale Reynolds number R_λ is approximately 1130 (675) in Series 1 (Series 2).

The analysis of the DNS data has shown new aspects of the small-scale statistics of turbulence, including (i) the normalized mean energy dissipation rate, (ii) the energy spectrum, (iii) the third order velocity structure function, (iv) intermittency of energy dissipation, and (v) the energy cascade from large to small scales.

A review is also given on a theory of the anisotropy in small-scale statistics in turbulent shear flow, stably stratified turbulence and magneto-hydrodynamic turbulence under a strong uniform magnetic field. A simple analysis suggests that at small scale, the effect of mean shear, buoyancy, and external magnetic field may be regarded as a disturbance applied to the locally homogeneous and isotropic universal equilibrium state. The theoretical predictions are in good agreement with DNSs.

2. ENERGY DISIIPATION AND SPECTRUM

One of the most characteristic features of high Reynolds number turbulence is the existence of a wide gap between the scales of the energy containing range and dissipation range. In our DNSs the ratio L/η of the integral length scale L to η is approximately 2130 (1040), in Series 1 (Series 2). The DNS data with such a wide separation may shed some light on fundamental questions on turbulence.

Among such questions is the one on the normalized dissipation rate $D=\bar{\varepsilon} L/U^3$, where $\bar{\varepsilon}$ is the average of the kinematic energy dissipation rate ε per unit mass, and $U^2/2$ is the total kinetic energy per unit mass of the fluctuating velocity field with zero mean. One may ask if D remains finite, or tends to 0, as $Re \to \infty$, i.e., the kinematic viscosity $\nu \to 0$, with U and L kept constant. It is one of the most fundamental assumptions in various theories of turbulence that it tends to a finite non-zero constant independent of *Re,* as $Re \to \infty$.

The data analysis by Kaneda et al.[5] showed that the DNS with R_λ up to approximately 1130 strongly supports this assumption. This implies that the turbulence with finite but very small ν is essentially different from the motion of ideal fluid in which there is no energy dissipation.

Another question of fundamental interest is on the energy spectrum $E(k)$, which is one of the most representative measures characterizing turbulence consisting of eddies over a wide scale range. One may ask if the DNS supports the prediction of K41 for the spectrum. According to K41, there is a universal equilibrium range of the wave number $k >> 1/L$, where $E(k)$ is a universal function of only k, ν, and $\overline{\varepsilon}$. In particular, in the inertial subrange $1/L << k << 1/\eta$, K41 gives the -5/3 law,

$$E(k) = K_o \overline{\varepsilon}^{2/3} k^{-5/3},$$

where K_o is a universal constant independent of *k*. The DNS data fit well to the -5/3 law with K_o =1.6~1.7. In the near dissipation range, where $k\eta$ ~1, the DNS data fit well to the form

$$E(k)/(\overline{\varepsilon}\nu^5)^{1/4} = C(k\eta)^a \exp[-b(k\eta)] \quad ,$$

where *C*, *a* and *b* are constants independent of *k*. The DNS data suggest that $(C, a, b) \to (0.044, -2.9, 0.62)$ as $R_\lambda \to \infty$. However, the approach is very slow; for example even at R_λ ~ 10,000, $|b(R_\lambda) - b(R_\lambda \to \infty)| / b(R_\lambda \to \infty)$ is as large as approximately 2.61.[3]

3. EXAMINATION OF KOLMOGOROV'S 4/5 LAW

Experiments and DNSs so far made generally support the idea of K41, at least for low order moments such as the energy spectrum and 2nd and 3rd order structure functions. It is however to be noted that the idea of K41 on the universality concerns with the asymptotic state at *Re* and $L/r \to \infty$. On the other hand, in any real turbulence and DNS, *Re* and *L/r* can be only finite, whatever large they may be. One may then ask if *Re* and *L/r* in the turbulence in one's consideration high enough or not for the universality. To answer this question, we need have quantitative understanding on the dependence of the statistics on *Re* and *L/r*.

In this respect, it may be instructive to consider the 4/5 law derived by Kolmogorov[10], according to which

$$< D_{LLL}(r) > \equiv < [\delta u_L(r)]^3 > = -\frac{4}{5}\overline{\varepsilon}\, r, \qquad (1)$$

in the inertial subrange of homogeneous isotropic turbulence, where $\delta u_L(r)$ is the longitudinal velocity difference at two points with distance *r*. The relation (1) is exact in the limit of $Re, L/r, r/\eta \to \infty$, and posses a unique position, since such an exact non-trivial relation is rare in turbulence study.

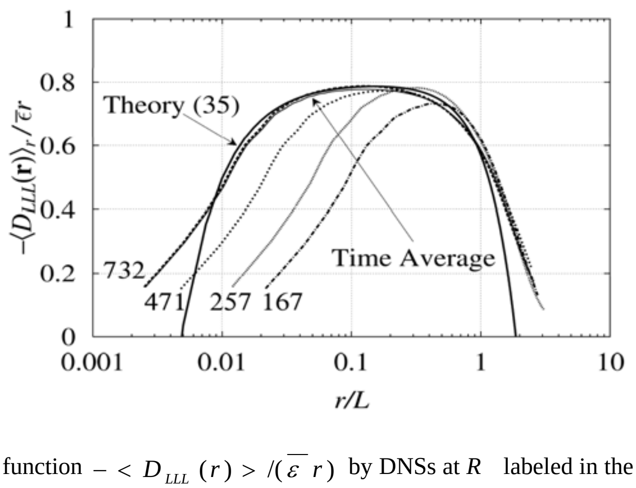

Fig.1. Normalized structure function $-< D_{LLL}(r) > /(\overline{\varepsilon}\, r)$ by DNSs at R labeled in the figure. The solid line is by a simple theory. From Ref. 8.

Figure1shows the DNS data for the normalized structure function $\overline{D}(r) \equiv - < D_{LLL}(r) > /(\overline{\varepsilon}\ r)$. As seen in Fig.1, $\overline{D}(r)$ is not exactly 4/5, in contrast to the theory (3), due to the finiteness of *Re*, *L/r* and *r/η*. The deviation $\delta(r) \equiv 4/5 - \overline{D}(r)$ is large for small *r* and large *r* . The range where $\overline{D}(r)$ is close to 4/5 and almost flat is wider for larger R_λ. An analysis shows that the deviation δ(*r*) consist of the contributions representing (i) viscous effect, (ii) external forcing, (iii) non-stationarity, and (iv) anisotropy of the statistics. In almost stationary turbulence with external forcing confined at large scale, the contribution of (iii) is negligible, and that of (ii) scales with *r* as $\propto (r/L)^2$, for small $(r/L)^2$. A simple theory shows that the contribution of (i) scales with *r* as $\propto (\eta/r)^{4/3}$ in the inertial subrange. A comparison of such a theory with the DNS data is shown in Fig. 1, and the theory is seen to be in good agreement with DNS.[8]

4. INTERMITTENCY OF ENERGY DISIIPATION

High *Re* turbulence exhibits strong intermittency at small scales. It is manifested at high order statistics. In order to explain the intermittency, Kolmogorov[11] introduced the kinetic energy dissipation rate $\varepsilon(r|\mathbf{x},t)$ at time *t* averaged over the sphere of radius *r* center at position $\mathbf{x}$. For ε_n defined by $\varepsilon_n = \varepsilon(r_n|\mathbf{x},t)$ with $r_n = r_0 a^n$, where *a* is a constant greater than 1, and $r_0=L$, the well-known decomposition

$$\frac{\varepsilon_n}{\varepsilon_0} = \frac{\varepsilon_n}{\varepsilon_{n-1}} \cdot \frac{\varepsilon_{n-1}}{\varepsilon_{n-2}} \cdots \frac{\varepsilon_1}{\varepsilon_0}, \tag{2}$$

gives

$$\log\frac{\varepsilon_n}{\varepsilon_0} = \log\alpha_{n-1} + \log\alpha_{n-2} + \cdots + \log\alpha_0, \quad \left(\alpha_n \equiv \frac{\varepsilon_{n+1}}{\varepsilon_n}\right). \tag{3}$$

The relations (2) and (3) suggest the importance of quantitative understanding on the statistics of the break down ratio α_n or equivalently log α_n for the understanding the statistics of the intermittency of ε.

There have been various studies of the intermittency by assuming certain statistical nature of α_n, or u_n =log α_n/log *a*. In such studies, it is often assumed that (i) u_n's (*n*=0,1,2, …) are statistically independent from each other, and/or (ii) the statistics of u_n is independent of *n* in the inertial subrange.

The analysis of the DNS data with R_λ up to 732 suggests that (i) the statistical dependence between u_n's has significant influence on the statistics of ε, and (ii) DNS does not support the conjecture of the existence of a scale range satisfying (ii), as seen in Fig.2.[6] But the analysis also suggests the existence of a range where the correlation between u_n and u_{n+m} is insensitive to *n*.

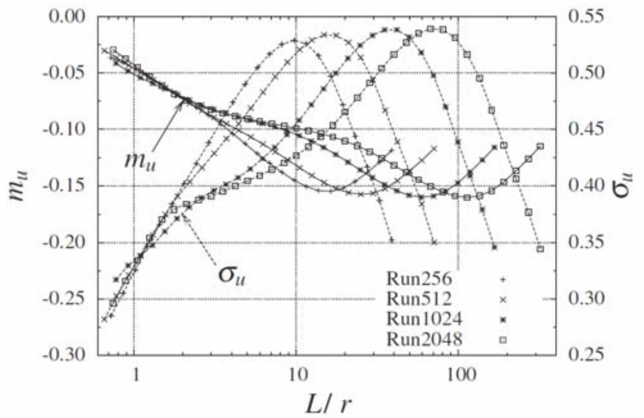

Fig.2. The mean m_u (solid lines, left scale) and the standard deviation σ_u (dotted lines, left scale) of u_n vs *L/r* with $r=r_n$ in a series of DNSs, Run256, 512,1024 and 2048, for which R_λ ~167,257,471 and 732, respectively. From Ref.6.

5. ENERGY TRANSFER

The statistics of the energy transfer from large to small scales is one of the most basic statistics characterizing the inertial subrange dynamics. Let *T*(*k*) be the energy transfer defined by $T(k) = -\Sigma_{ij}\tau_{ij}\overline{S_{ij}}$ with $\tau_{ij} = \overline{v_i v_j} - \overline{v}_i\overline{v}_j - (2/3)\delta_{ij}q,\ \ q = (1/2)\Sigma_k(\overline{v_k v_k} - \overline{v}_k\overline{v}_k)$, $S_{ij} = (1/2)(\partial\overline{v}_i/\partial x_j + \partial\overline{v}_j/\partial x_i)$,

v_i is the *i*-th Cartesian velocity component, $\overline{f}$ is the grid scale component of f, and we use the so-called spectral cut-off filter to define $\overline{f}$, by which all the Fourier modes of f with wave number larger than the cut-off wave number k are removed.

The DNS data shows that $T(k)$ is highly intermittent and the skewness S and the flatness factor F of $T(k)$ scale with k like $S \propto (kL)^a$ and $F \propto (kL)^b$, where a~2/3 and b~1.[1] The comparison of the statistics of $T(k)$, ε and $\varepsilon(r|\mathbf{x},t)$ shows that $T(k)$ is less intermittent than ε, but there is a certain similarity between the probability distribution function of $T(k)$ and $\varepsilon(r|\mathbf{x},t)$, where $k= \pi/(2r)$.[1]

6. UNIVERSALITY OF THE SECOND KIND

The statistical mechanics of systems at or near thermal equilibrium presents a paradigm of the study of systems with a huge number of degrees of freedom (DOF). In the study, it is known that there are two kinds of universal relations; one is those characterizing the equilibrium itself, and the other is those characterizing the responses to disturbances added to the equilibrium system. We call here the latter "universality of the second kind". The relations belonging to the latter may be written symbolically as $J=CX$, where the generalized flux and force (J, X), which represent the response and the disturbance respectively, may be (scalar gradient, density flux), (temperature gradient, heat flux), (electric field, electric current), etc. The coefficient C reflects the equilibrium state of the system.

Turbulence is also a phenomenon involving a huge number of DOF, and it is tempting to assume that there are two kinds of universal relations also in turbulence statistics. This idea was applied to turbulent shear flow by Ishihara et al.[4]. It is shown that in the inertial subrange, the turbulence is dominated by the inherent Navier-Stokes dynamics without mean shear, and the effect of mean shear may be regarded as a perturbation added to the universal equilibrium state. The theoretical conjectures were shown to be in good agreement with high-resolution DNSs of turbulence under a mean flow of a simple shearing motion. The idea was also applied to strongly stably stratified incompressible turbulence obeying the Boussinesq equation[7], and also to magneto-hydrodynamic turbulence under strong uniform magnetic field obeying the so-called adiabatic equations[2]. The idea was confirmed to be consistent with DNS.

ACKNOWLEDGMENT

This work was partially supported by Grant-in-Aid (B)20340099 from the Japan Society for the Promotion of Science.

REFERENCES

[1] Aoyama T, Ishihara T, Kaneda Y. Statistics of energy transfer in high-resolution direct numerical simulation of turbulence in a periodic box. J. Phys. Soc. Japan 2005, 74 (12), 3202-3212

[2] Ishida T and Kaneda Y. Small-scale anisotropy in magnetohydrodynamic turbulence under a strong uniform magnetic field. Physics of Fluids 2007, 19(7), 075104

[3] Ishihara T, Kaneda Y, Yokokawa M, Itakura K, Uno A. Energy spectrum in the near dissipation range of high resolution direct numerical simulation of turbulence. J. Phys. Soc. Japan 2005, 74(5), 1464-1471

[4] Ishihara T, Yoshida K, Kaneda Y. Anisotropic velocity correlation spectrum at small scales in a homogeneous turbulent shear flow. Phys. Rev. Lett. 2002, 88(15), 154501

[5] Kaneda Y, Ishihara T, Yokokawa M, Itakura K, Uno A. Energy dissipation rate and energy spectrum in high resolution direct numerical simulation of turbulence in a periodic box. Phys. Fluids 2003, 15(2), L21-24

[6] Kaneda Y and Morishita K. Intermittency of energy dissipation in high-resolution direct numerical simulation of turbulence. J. Phys. Soc. Japan 2007, 76 (7), 073401

[7] Kaneda Y and Yoshida K. Small-scale anisotropy in stably stratified turbulence, New J. Physics 2004, 6, 34

[8] Kaneda Y, Yoshino J, Ishihara T. Examination of Kolmogorov's 4/5 law by high-resolution direct numerical simulation data of turbulence. to appear in J. Phys. Soc. Japan.

[9] Kolmogorov AN. The local structure of turbulence in incompressible viscous fluid for very large Reynolds number. Dokl. Akad. Nauk SSSR 1941, 30(4), 301-305

[10] Kolmogorov AN. Dissipation of energy in locally isotropic turbulence. Dokl. Akad. Nauk SSSR 1941,32(1), 16-18

[11] Kolmogorov AN. A refinement of previous hypotheses concerning the local structure of turbulence in a viscous incompressible fluid at high Reynolds number. J. Fluid Mech. 1962, 13(1), 82-85

[12] Yokokawa M, Itakura K, Uno A, Ishihara T, Kaneda Y. 16.4-Tflops direct numerical simulation of turbulence by a Fourier spectral method on the Earth Simulator. *Proceedings of IEEE/ACM SC2002 Conf.*, Baltimore, 2002; http://www.sc-2002.org/paperpdfs/pap.pap273.pdf.