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Modelling of High-Speed Turbulent and Turbulent-Transition Flows with RANS Approach

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ABSTRACT: *This article provides a brief review of turbulence modeling work at Tsinghua University over the last few years. Particular attention is concentrated on the three area: a) modelling of hypersonic boundary-layer transition flows; b) Modelling of the compressibility effect with second-moment closure; c) Modelling the rapid pressure-strain correlation satisfying RDT.*

1. INTRODUCTION

High-speed turbulence flows in engineering applications are often characterized with two fundamental flow features: the effect of compressibility and the effect of large velocity gradients. It is well known that compressibility effect plays an important role in determining the turbulence physics when Mach number Ma is high and the conventional Mokovian hypothesis leads to significant errors in modelling. It is also known that the compressibility strongly influences the behaviour of supersonic/hypersonic flow transition compared to low-speed case. The large velocity gradients in the flow, either in form of shear or strain, generally lead to rapid flow distortion to turbulence that requires delicate modelling practice. However, current compressible turbulence/transition models are still not able to provide satisfactory results to meet the urgent engineering demands. Existing turbulence models also fail to capture the physics of rapidly distorted turbulent flows which, in the limit, satisfy Rapid Distortion Theory (RDT).

This article provides an overview of some of the research efforts in turbulence modelling made at Tsinghua University over the last few years in an attempt to improve predictive capabilities for flows associated with significant compressibility and distortion effects. In particular, three area will be primarily focused:

- Modelling of hypersonic boundary-layer transition flows;
- Modelling of the compressibility effect with second-moment closure;
- Modelling the rapid pressure-strain correlation satisfying RDT.

2. MODELLING OF HYPERSONIC BOUNDARY-LAYER TRANSITION FLOWS

Modelling of flow transition has always been a research focal point in turbulence study. Currently, the RANS approach is still the main tool in the transition/turbulence modelling in engineering application. Since it is proved that turbulence model without making use of the intermittency are often extremely unreliable in the prediction of transition [1], there appear many correlation-based transition models involving the intermittency factor. However, these models include non-local formulations which are not easily compatible with modern CFD methods. The models based on local variables are thus much preferred for the application purpose. A successful example is the work of Menter et al. [2], which is now implemented in a commercial software package.

However, the existing local-variable-based models are not validated for the transition in supersonic flows or for the cross-flow transition. One reason is that these models rely on heavy load of numerical validation rather than the fundamental physical phenomena responsible for actual transition process, e.g. the flow instability mode can be rather different in supersonic boundary layers than that in incompressible or subsonic flows. The purpose of this investigation is to develop an improved flow transition model applicable to supersonic as well as three-dimensional flows.

Thus, a transition model based on k-ω-γ transport equations is proposed here. The model converts to the standard SST model [3] in the fully turbulent region. The fluctuating kinetic energy k includes the non-turbulent, as well as turbulent parts. The intermittency factor, γ, is set to play as a weight number between the non-turbulent and the turbulent components of stress in P_k and P_ω, i.e. the production terms of equations for k and ω. This approach focuses on the determination of effective viscosity of non-turbulent fluctuations, μ_{nt}, as derived from the linear stability theory (LST).

Both the LST and experimental observations give that at low Mach number the so-called 'first-mode' disturbance is the primary cause of instability while the effect of 'second-mode' disturbances becomes prominent at high Mach number flows. This mode variation, related to the effect of compressibility, is accommodated inherently in the present model through the local relative Mach number, i.e. $M_{rel} = (U - c_r) / a$, where U stands for the local mean velocity related to wall, a is the local sound speed, and c_r represents the phase velocity, as the same value, of all Mack-mode disturbances. μ_{nt} is determined by the timescale of the first-mode fluctuations at $M_{rel}^2 \leq 1$ while both of the first-mode and second-mode ones at $M_{rel}^2 > 1$.

According to the experimental correlations and theoretical analysis, the formulations of μ_{nt} would involve non-local variables, such as the boundary layer thickness, which is calculated from integrals through the boundary layer. To avoid this practice, this study defines a length scale normal to wall of mean flow as $d^2 \Omega / (2\ Eu)^{0.5}$, where d is the distance to wall, Ω is the absolute value of mean vorticity, and E_u stands for the kinetic energy of mean flow.

Moreover, a new transport equation for intermittency factor is developed. Its particular feature is that a function in the source term, closely related to the flow physics, is set to trigger the onset of transition. The present model proposal is calibrated and validated with three sets of experimental data involving incompressible flow over a flat plate, supersonic flow past a straight cone and hypersonic flow over a flared cone at zero angle of attack. The skin frictions for T3A flat-plate test case (http://cfd.me.umist.ac.uk/ercoftac/) is shown in Fig.1. It is seen that the flow transition profile are well captured with Menter's model [2] and the present model. Fig.2 and Fig.3 compare the measured and computed recovery factor and wall temperature distributions for cones in high-speed flows, respectively. The present model gives accurate transition onsets but misses peak values. For latter case, Hassan's model [4] gives too low temperature level though the onset location seems not bad.

In conclusion, a new k-ω-γ transition/turbulence model considering the modes of instability is proposed and validated in this work. It is based on local variables and is able to trigger the onset of transition automatically with the function in the source term of γ equation. The present model has been successfully applied to simulate the natural, as well as the bypass transitions.

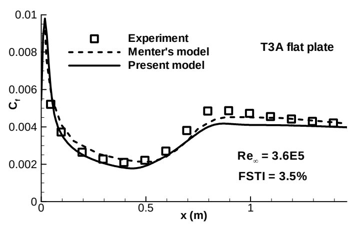

Figure 1: Skin friction (C_f) for the T3A test case. FSTI represents the free-stream turbulence intensity .

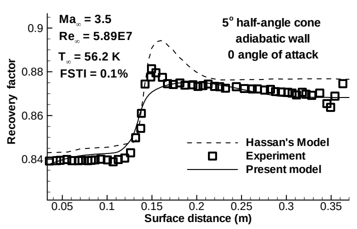

Figure 2: Comparison of computed and measured recovery factor.

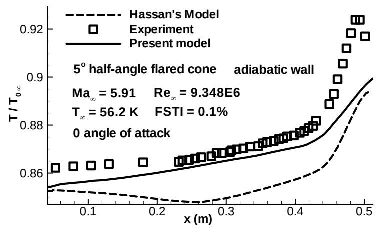

Figure 3: Comparison of computed and measured wall temperature distribution.

3 MODELLING OF THE COMPRESSIBILITY EFFECT WITH SECOND-MOMENT CLOSURE

It is often observed from the DNS and experiment results of compressible mixing layers that the Reynolds stresses decrease with the increased convection Mach number (M_c). However, different observations for the behaviours of the Reynolds stress anisotropy have also been made. If one looks into the discrepancy of the shear stress anisotropy, the data can be seen to fall into two classes. The shear stress anisotropy remains relatively unaffected as shown by the data [5-8], while the shear stress anisotropy decreases with increasing compressibility effects as shown by the other data [9-12]. Here we try to find the cause of the discrepancy.

First, the maximum values of the shear stress anisotropy and the shear stress correlation coefficient as functions of M_c are plotted in Fig.4. Here, three different regions characterizing the compressibility effect are seen to exist. In region 1 ($0<M_c<1$) the variation in the shear stress anisotropy b_{12} and the shear stress correlation coefficient $D \equiv \langle u''v''\rangle / \sqrt{\langle u''u''\rangle\langle v''v''\rangle}$ is small and can be considered roughly constant; in region 2 ($1<M_c<1.5$) b_{12} declines linearly with M_c; in region 3 ($M_c > 1.5$) b_{12} reaches a constant value again. These classification, although a rather simplified view, suggest that the discrepancy of the shear stress anisotropy behaviour existing data may be related to the different values in their corresponding M_c.

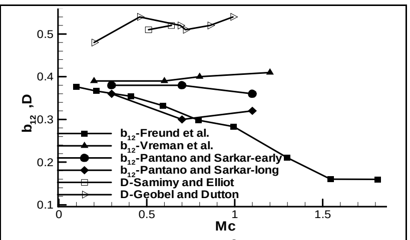

Figure 4: Variation of the shear stress anisotropy and correlation coefficient

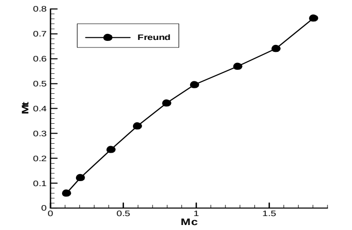

Figure 5. Variation of the centerline turbulent Mach number

Also, it can be illustrated that M_c value is strongly correlated with the turbulent Mach number M_t. Fig.5 shows the variation of the centerline M_t with M_c in the DNS data of Freund *et al*. [8]. Here, it is seen that M_c, corresponding to M_t 0.4, is very near 1, M_c corresponding to the turbulent Mach number M_t 0.6 is very near 1.5. Thus, in view of M_t the three regions can be divided into: $M_t<0.4$, $0.4< M_t<0.6$ and $M_t>0.6$. The use of M_t allows further analysis in terms of M_t rather than M_c for the mixing layer flows.

Based on the above analysis, a new compressible second-moment model using the idea of zonal effects is developed and successfully applied to the mixing layer calculations.

Basic Ideas of the New Compressible Model

Previous studies focused on the dilatational effects of velocity fluctuation. These dilatational terms appear explicitly in the turbulent kinetic energy equation, offering to reduce the growth rate of compressible mixing layers. However, they do not alter the Reynolds stress anisotropy significantly, because the normal stress is reduced in an isotropic manner [13]. Recent studies focused on the pressure-strain term which successfully reduces the growth rate of compressible mixing layers through reduced turbulence production. A simple but effective pressure-strain model is proposed here reflecting the compressible effect.

However, the models proposed for the dilatation reduce the growth rate of the compressible mixing layer while not affecting the Reynolds stress anisotropy significantly. This is the case in the low-M_t region of the compressible mixing layer mentioned above. The models proposed for the pressure–strain term reduce the growth rate of the compressible mixing layer and alter the behaviour of the Reynolds stress anisotropy. This is what we find in the transition-M_t region mentioned above. These give us a key idea to develop a new model.

Closure for the Dilatational Terms and Pressure–Strain Term

Since the dilatation is the main effect in the *low-M_t* region and the closure for the dilatational terms should be effective mainly in this region. Following the idea of Sarkar *et.al* [14], we propose the following algebraic model for the dilatation–dissipation term (ε_d):

$$\varepsilon_d = \chi \cdot \varepsilon_s ; \qquad \chi \equiv 0.1 \cdot \left[1 - \exp\left(-5M_t^2\right)\right]; \tag{1}$$

This makes the model consistent with the model proposed by Sarkar *et al*. [11] for low M_t but more accurate for high M_t flows. It will make the dilatation–dissipation term dominate in the low-M_t region, while small in other regions.

For the simplicity, the model proposed by Gibson & Launder [15] is chosen as the base model for the compressible rapid pressure–strain term. However, the effect of model coefficients as functions of M_t is investigated. The following form for the rapid pressure–strain model is found appropriate:

$$\Pi_{ij} = -C_1 a_{ij}\varepsilon + C_2 k\left(S_{ij} - 1/3 S_{ll}\delta_{ij}\right) + C_3 k\left(a_{il}S_{lj} + S_{il}a_{lj} - 2/3 a_{kl}S_{lk}\delta_{ij}\right) + C_4 k\left(W_{il}a_{lj} - a_{il}W_{lj}\right) \tag{2}$$

where, C_1=1.8, C_2=0.8, C_3=$0.6 + F(M_t)$, C_4=$0.6 - F(M_t)$. When $F(M_t) = 0$, the model returns to the original incompressible version of the Gibson & Launder's model [15]. Following the zonal idea, the effect of compressibility should quickly increase in the transition-M_t region. So, we set:

$$F(M_t) \equiv 0.25 \cdot \exp\left(-0.05/M_t^3\right) \tag{3}$$

It is seen here that the function $F(M_t)$ is near zero for low turbulent Mach numbers that will keep the shear stress anisotropy unaffected in the low-Mt region. In transition-M_t region the function will increase quickly with increasing M_t, finally the function reach a constant in high-M_t region. These ensure the effect of the pressure–strain term changing as we expect.

Application of the Model

The new turbulence model has been applied to the simulation of compressible mixing layers. Figure 6 shows the variation of the peak turbulent Mach number with the convective Mach number. Freund *et al.*'s [8] DNS results are also included. As can be seen, the result of the present model is slightly below the Freund *et al.*'s data. Finer tuning of the coefficient functions may be required in future work. But the trend is satisfactory. Figure 7 shows the variation of the peak shear stress anisotropy. As expected, the variation of the present model's shear stress anisotropy clearly follows the existing data. The shear stress anisotropy is nearly unchanged in the low-M_t region; and after a quickly change in the transition-M_t region. The calculated growth rates agree reasonably well with the 'Langley curve'.

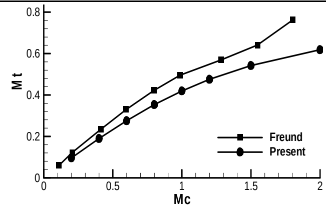

Figure 6. Variation of the peak turbulent Mach number.

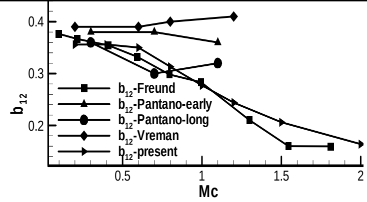

Figure 7. Variation of the peak shear stress anisotropy.

4. MODELLING THE RAPID PRESSURE-STRAIN CORRELATION WITH RDT

When turbulence is subjected to rapid distortion, the description of turbulence evolution simplifies significantly, leading to the so-called rapid-distortion theory (RDT) equations [16]. Larsson's DNS data [17] show, gradually increasing the mean strain rate, DNS should give results converging towards the RDT solution. Lee *et al.* [18] shows RDT may give an accurate description of the turbulence at realistic strain rates, and they pointed out that RDT contains the essential mechanism responsible for development of turbulence structures in the presence of high shear rate. These make RDT results become important references in rapid pressure-strain models proposals. We believe that a good turbulence model should match RDT for rapid deformations. However, later we will show the current models give very poor results in high strain rate situation. In the present paper we will analyze the causes for the current rapid pressure-strain models deficiencies, and present a new rapid pressure-strain model by expanding the classical assumption of M-tensor.

Analyses of deficiencies in classical rapid pressure-strain models

First, let's scrutinize the transport equation of homogeneous turbulence a little further for Reynolds-stress anisotropy tensor b_{ij} that can be written as (in RDT limit, the effects of slow pressure-strain correlation and dissipation are absent):

$$db_{ij}/dt = P_{ij}^{b} + \Phi_{ij}^{br} = T_{ij} \qquad (4)$$

Johansson & Hallbäck [19] derived the most general expression for the rapid pressure-strain models in the context of the current approach for the Reynolds stress models.

$$T_{ij} = [q_1 \mathrm{I}_{bS} + q_9 \mathrm{I}_{bbS}]G_{ij}^{(1)} + q_2 G_{ij}^{(2)} + q_3 G_{ij}^{(3)} + q_4 G_{ij}^{(4)} + (q_5 \mathrm{I}_{bS} + q_{10}\mathrm{I}_{bbS})G_{ij}^{(5)} + q_6 G_{ij}^{(6)} + q_7 G_{ij}^{(7)} + q_8 G_{ij}^{(8)} \qquad (5)$$

$$\left.\begin{array}{llll} G_{ij}^{(1)} = b_{ij} & G_{ij}^{(3)} = b_{ik}S_{kj} + S_{ik}b_{kj} - 2\mathrm{I}_{aS}\delta_{ij}/3 & G_{ij}^{(5)} = b_{ik}b_{kj} - \mathrm{II}_b\delta_{ij}/3 & G_{ij}^{(7)} = b_{ik}b_{kl}\Omega_{lj} - \Omega_{ik}b_{kl}b_{lj} \\ G_{ij}^{(2)} = S_{ij} & G_{ij}^{(4)} = b_{ik}\Omega_{kj} - \Omega_{ik}b_{kj} & G_{ij}^{(6)} = b_{ik}S_{kl}b_{lj} - \mathrm{I}_{bbS}\delta_{ij}/3 & G_{ij}^{(8)} = b_{ik}b_{kl}\Omega_{lm}b_{mj} - b_{im}\Omega_{mk}b_{kl}b_{lj} \end{array}\right\}$$

where $\mathrm{I}_{bS} = b_{kl}S_{lk}$, $\mathrm{I}_{bbS} = b_{kl}b_{lm}S_{mk}$, the q_i, may depend on the second and third invariants of b_{ij}, namely $\mathrm{II}_b = b_{kl}b_{lk}$ and $\mathrm{III}_b = b_{kl}b_{lm}b_{mk}$. Any dependence on the Reynolds number is not relevant in the rapid distortion limit.

The transport equations for the invariants II_b and III_b in RDT limit can be derived from Equations (4) and (5), through some elaborate tensor algebra, that finally lead to

$$\begin{aligned} d\mathrm{II}_b/dt &= \left[2q_2 + (2q_1 + q_6)\mathrm{II}_b + 2q_5\mathrm{III}_b\right]\mathrm{I}_{bS} + \left[4q_3 + 2q_9\mathrm{II}_b + 2q_{10}\mathrm{III}_b\right]\mathrm{I}_{bbS} \\ d\mathrm{III}_b/dt &= \left[q_3\mathrm{II}_b + q_5\mathrm{II}_b^2/2 + \left[q_6 + 3q_1\right]\mathrm{III}_b\right]\mathrm{I}_{bS} + \left[3q_2 + (q_{10}\mathrm{II}_b^2 + q_6\mathrm{II}_b)/2 + 3q_9\mathrm{III}_b\right]\mathrm{I}_{bbS} \end{aligned} \qquad (6)$$

In pure rotation ($S = 0$), we easily get

$$d\,\mathrm{II}_b/dt = 0,\ d\mathrm{III}_b/dt = 0. \tag{7}$$

So, current model predictions fail to give damped oscillations in the turbulence anisotropy. The cause can be attributed directly to the absence of the rotation term Ω in Eq. (6).

To further elucidate the implication of Equations (6), it is appropriate to consider the behaviour of these two equations in the case of homogeneous two-dimensional mean flows. For initially isotropic turbulence subjected to two-dimensional mean flows in RDT limit, Reynolds stresses will have one principal axis that is always perpendicular to the plane of the flows. We set the eigenvalues of the principal axis to be b_{33} (b_{33} is thus an invariant also, and $b_{23}=0$, $b_{31}=0$). In this case, we are easy to get $\mathrm{I}_{bbS}/\mathrm{I}_{bS} = -b_{33}$. Then, the time evolution for this component is given as

$$db_{33}/dt = [q_1\mathrm{I}_{bs} + q_9\mathrm{I}_{bbS}]a_{33} + q_3(-2\mathrm{I}_{bS}/3) + [q_5\mathrm{I}_{bS} + q_{10}\mathrm{I}_{bbS}](b_{33}^2 - \mathrm{II}_b/3) + q_6(-\mathrm{I}_{bbS}/3) \tag{8}$$

Now we can write Equations (3), and (5) in more simple form

$$\left.\begin{aligned} &d\mathrm{II}_b/(\mathrm{I}_{bS}dt) = d\mathrm{II}_b/d\tau = (f_1(\mathrm{II}_b,\mathrm{III}_b) - f_2(\mathrm{II}_b,\mathrm{III}_b)b_{33});\quad d\mathrm{III}_b/\mathrm{I}_{bS}dt = d\mathrm{III}_b/d\tau = (g_1(\mathrm{II}_b,\mathrm{III}_b) - g_2(\mathrm{II}_b,\mathrm{III}_b)b_{33}) \\ &db_{33}/\mathrm{I}_{bS}dt = db_{33}/d\tau = (h_1(\mathrm{II}_b,\mathrm{III}_b) - h_2(\mathrm{II}_b,\mathrm{III}_b)b_{33} - h_3(\mathrm{II}_b,\mathrm{III}_b)b_{33}^2 - h_4(\mathrm{II}_b,\mathrm{III}_b)b_{33}^3) \end{aligned}\right\} \tag{9}$$

It is important to note that the Equations (9), representing a dynamic system of order three, are self-closed in terms of function of II_b , III_b and b_{33} in two-dimensional mean flows. Thus, all two-dimensional flows will have the same path on the AIM.The above analysis shows two important deficiencies in the current rapid pressure-strain models, no matter how high and how complex the expansion in the Reynolds-stress anisotropy tensor is. The cause of the problem may connect with the absence of the strain rate and rotation rate tensors, S and Ω , in Eq (6) which are insensitive to flows.

A New Proposal for the Rapid Pressure-Strain Model

As discussed above, we think the current expansion of M-tensor is insufficient and should include the effect of strain and rotation rate tensors. Although the mean-flow quantities do not appear directly in the M-tensor expression, the two-point nature of the correlations of the fluctuating velocity gradients suggests that they are inherently related to the mean velocity gradients. While M-tensor involves information that is not contained in Reynolds-stress anisotropy tensor [20], mean-flow quantities can affect M-tensor by change the lost information. This is likely true in many cases, a nonlinear expression in the mean velocity gradient should be considered more general. In the present work, the nonlinearity in the mean velocity gradients is considered in the M-tensor. Here we assume,

$$M_{likj} = M^{b}_{likj}(b) + M^{\Omega b}_{likj}(b,\Omega^*),\quad \Omega^* = (0.5\cdot(U_{i,j} - U_{j,i}))/\sqrt{U_{l,m}U_{l,m}} \tag{10}$$

$M^{b}_{likj}(b)$ is expansion of tensor b alone. This makes the new model easy to combine with current turbulence theory. In this paper, we select the form of FLT model [21] to displace $M^{b}_{likj}(b)$. $M^{\Omega b}_{likj}(b,\Omega^*)$ is an expansion of tensor b and Ω^*. $M^{\Omega b}_{likj}(b,\Omega^*)$ can be seen as a correction term of $M^{b}_{likj}(b)$. Here we select quadratic expansion form of $M^{\Omega b}_{likj}(b,\Omega^*)$,

$$\begin{aligned} M^{\Omega b}_{\mathrm{likj}} &= \alpha_1(\Omega^*{}_{lj}\delta_{ki} + \Omega^*{}_{ij}\delta_{kl} + \Omega^*{}_{lk}\delta_{ji} + \Omega^*{}_{ik}\delta_{jl}) + \alpha_2(\Omega^*b - b\Omega^*)_{li}\delta_{kj} + \alpha_3(\Omega^*b - b\Omega^*)_{kj}\delta_{li} + \alpha_4((\Omega^*b)_{lj}\delta_{ki} + (\Omega^*b)_{ij}\delta_{kl} \\ &+ (\Omega^*b)_{lk}\delta_{ji} + (\Omega^*b)_{ik}\delta_{jl}) + \alpha_5((b\Omega^*)_{lj}\delta_{ki} + (b\Omega^*)_{ij}\delta_{kl} + (b\Omega^*)_{lk}\delta_{ij} + (b\Omega^*)_{ik}\delta_{lj}) + \alpha_6(\Omega^*{}_{lj}b_{ki} + \Omega^*{}_{ij}b_{kl} + \Omega^*{}_{lk}b_{ji} + \Omega^*{}_{ik}b_{jl}) \\ &+ \alpha_7\Omega^{*2}{}_{kj}\delta_{li} + \alpha_8\Omega^{*2}{}_{li}\delta_{kj} + \alpha_9(\Omega^{*2}{}_{lj}\delta_{ki} + \Omega^{*2}{}_{ij}\delta_{kl} + \Omega^{*2}{}_{lk}\delta_{ji} + \Omega^{*2}{}_{ik}\delta_{jl}) + \alpha_{10}(\Omega^*{}_{lj}\Omega^*{}_{ki} + \Omega^*{}_{ij}\Omega^*{}_{kl}) + \alpha_{11}\Omega^{*2}{}_{nn}\delta_{li}\delta_{kj} + \alpha_{12}\Omega^{*2}{}_{nn}(\delta_{lj}\delta_{ki} + \delta_{lk}\delta_{ij}) \end{aligned} \tag{11}$$

The coefficients in the above relation are determined in the same manner as FLT.

Because of the deficiencies mentioned above, the current rapid pressure-strain models work very poorly in RDT limit. By comparison in figure 8-10 (where “present model” is result of this work, “RDT” is result of RDT, “GL” is result of Gibson-Launder model, “SSG” is result of SSG model), the results of present model is satisfied.

A new approach to improving the prediction of the anisotropy evolution in rapid limit, where the M - tensor of rapid pressure-strain correlation is expandable in the Reynolds-stress anisotropy tensor and mean rotation rate tensor. This extension allows two different traces in AIM for plane strain and shear flow, and a damped oscillatory solution for the anisotropy components in the situation of pure rotation, which are not even qualitative captured by classical rapid pressure-strain models. Present work presents a possible way to extend classical model to rapid deformations field.

ACKNOWLEDGMENTS

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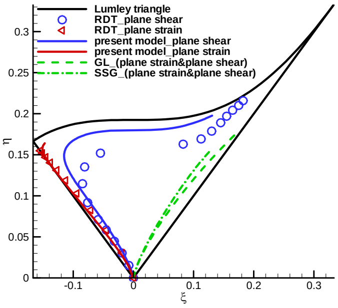

Figure 8. AIM paths in homogeneous plane strain and shear

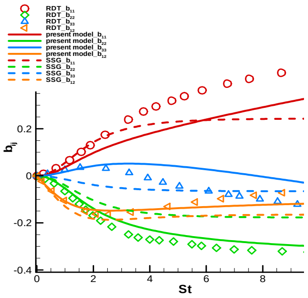

Figure 9. Reynolds-stress anisotropies for shearflow

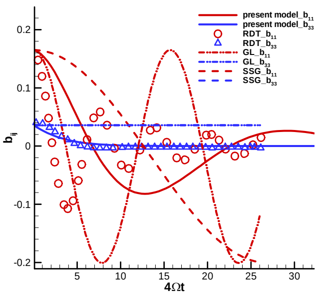

Figure 10. Reynolds-stress anisotropies for pure rotation flow

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