Laminar-turbulent transition and turbulence modeling of supersonic and hypersonic boundary layers

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Abstract Two problems will be addressed in this paper, namely the prediction of laminar-turbulent transition and the computation of supersonic and hypersonic turbulent boundary layers. For the former, the aim is to make the conventional e^N method more rational, i.e, greatly reduce its dependence on experiment and experience. For the latter, we will show that for some popular turbulence models, such as k-ε, k-ω, and SST models, the fundamental assumption may not valid. On the other hand, ways for the improvement on BL model are proposed.

1. Introduction

The prediction of laminar-turbulent transition and the computation of turbulent flows are two most difficult problems in fluid mechanics, either from theoretical or practical application point of view, especially for hypersonic, or even supersonic flows. In [1], J. J. Bertin and R. M. Cummings have listed several factors affecting the accuracy of flow predictions for re-entry vehicles, among them, the ability to model real-gas effect and to model transition and turbulence are the factors related to gas dynamics. In the same paper, Bushnell was quoted to saying that "historically, man has been singularly unsuccessful in 'predicting' transition on essentially everything flown hypersonically (or even supersonically)."

In recent years, we have made some progresses concerning the transition prediction and the turbulence modeling, which will be briefly reported in this paper.

2. Laminar-turbulent transition prediction

Fig. 1 shows the scenario of laminar-turbulent transition of a boundary layer on a flat plate, from which one can see that there are three questions that one has to address in order to make the transition predict rational, namely, how the disturbances in the boundary layer are excited, how the disturbances evolve, and at which stage of the disturbance evolution the transition will be triggered.

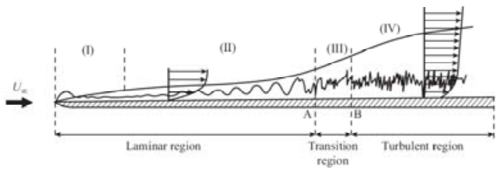


Fig. 1 Sketch of the boundary layer problem

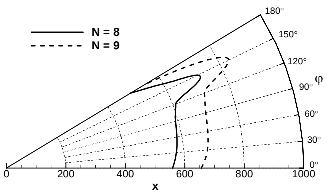
Currently, the most popular method of transition prediction is the e^N method, which is essentially a semi-empirical method, heavily relies on experiments and experiences. To make it more rational is highly desirable. For natural transition, which starts with small amplitude disturbances, in most part of laminar region, linear stability analysis is usually sufficient to deal with the evolution problems of the disturbances. Therefore, what we proposed to make it more rational consist essentially two aspects: 1. Incorporate results from receptivity studies to specify the initial disturbances. 2. Based on DNS results of transition studies, instead of specifying N, a criterion for transition based on the amplitude of disturbances is proposed. In applying this improved method for the transition prediction of supersonic and hypersonic boundary layers on a cone with small angle of attack, results are very encouraging.

For a cone with small bluntness, flying in a quiet environment, we assume the disturbances mainly come into the boundary layer through their interaction with the mean flow near the stagnation point, just as the receptivity mechanism at the attachment line for an air foil, so at a location not too far downstream of the spherical head, all disturbances, with different frequencies, have nearly the same initial amplitude. Start from that location, not from the ZARF as in the conventional e^N method, we use the method of linear stability analysis, just as in the e^N method, to trace the evolution of the amplitudes of the disturbance.

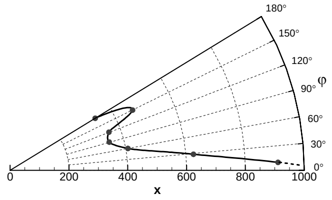
In conventional e^N method, transition location is determined by the factor N, which must be determined from experiments or experiences. But from results of DNS done by us for the transition of boundary layers, including incompressible, supersonic, as well as hypersonic flows, we concluded that transition would take place whenever the amplitude of disturbances reaches $0.01 \sim 0.02$ of the free stream velocity. Therefore, instead of specifying N, we use the criterion based on the amplitude of the disturbances. In fact, the difference resulting from choosing the critical amplitude to be 0.01 or 0.02 is equivalent to the difference resulting from choosing the factor N with uncertainty 0.7.

In this way, the prediction of the transition location no longer relies heavily on experiments or experience, the only uncertainty, or adjustable parameter is the guessed initial amplitude of the disturbance.

Fig.2 shows the results of transition prediction for a cone with small bluntness and angle of attack 1° at Ma=6. Fig.2(a) is by the conventional e^N method, which is not reasonable, while Fig.2(b) is by the present method, Fig.2(c) is a comparison between results of our method and results from DNS, with initial disturbances consisting many waves, with equal amplitudes, in the form of blow and suction at a fixed location. The comparison is very satisfactory, and results of DNS did confirm that just before transition, the amplitude of the disturbances were within the rage of $0.01 \sim 0.02$.



(a)



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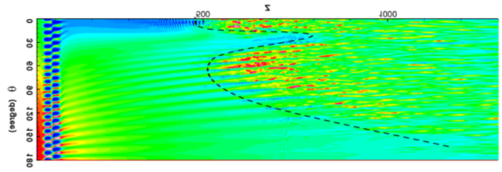
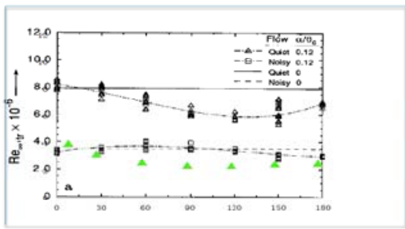


(c)

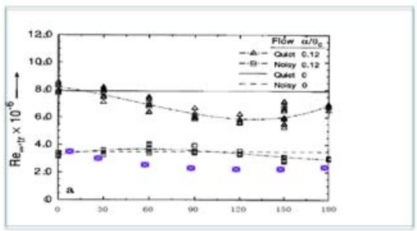
Fig.2 Transition location predicted by different methods

For more detail and more examples, see [2,3].

There is another kind of result from King's experiments [4]. He conducted transition experiments in a wind tunnel, which can be made either quiet or noisy, depending on the treatment of the boundary layer of the wind tunnel. The above transition prediction method can yield result agree well with those obtained under quiet condition, but can not yield result agree well with those obtained under noisy condition. Fig.3(a) shows the result (the green solid triangles) from the conventional e^N method after adjusting the value of N, and Fig3(b) shows the result (the purple rectangles) from the improved e^N method, also after properly adjusting the initial amplitude of disturbances. Both results do not yield the correct trend (their convexity) of the transition location.



(a)



(b)

Fig.3 Comparison of the transition location

Under noisy condition, the background disturbances are mainly acoustic waves generated by the turbulent boundary layer of the tunnel. Thus, the receptivity mechanism should be the receptivity to the acoustic wave. Any acoustic wave, interacting with the shock wave of the cone, will generate both fast and slow acoustic waves behind the shock. However, since the fast acoustic wave's phase speed does not match the phase speed of the T-S waves in the boundary layer, it can not excite the T-S wave directly. Although it can excite T-S waves indirectly through exciting first a certain kind of damping wave, its efficiency is greatly reduced. On the other hand, the slow acoustic wave's phase speed can match the phase speed of a certain T-S wave, hence can excite directly T-S waves. So, in applying the e^N method to this problem, we again do not start the linear stability analysis of the T-S waves from the ZARF, but start from the place where the phase speed of the slow acoustic waves match the T-S waves. Moreover, we have to properly adjust the initial amplitude of disturbances, and also properly considering the dependence of the initial disturbance amplitudes on their frequencies by referring to result of wind tunnel calibration (not King' tunnel, because he did not give result of calibration for his tunnel), then we obtained the result shown in Fig 4. We can see that at least the trend of the transition location (red diamonds) agrees well with the experimental result.

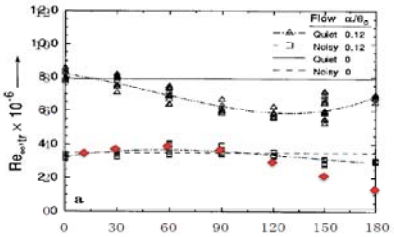


Fig. 4 Result by considering the receptivity on sound waves

The conclusion is: the conventional e^N method can be made much more rational, i.e. no longer depends heavily on experiments and experiences. For which one has to take into account the receptivity mechanism, as well as to use the amplitude of the disturbances, not the N factor, as the criterion of transition. The only parameter remains to be determined or guessed is the initial amplitude of the disturbances.

3. On turbulence modeling for supersonic and hypersonic turbulent boundary layers

Most existing turbulence models for supersonic and hypersonic turbulent boundary layers are borrowed from those for incompressible flows with some modifications. The fundamental difficulty for turbulence modeling of supersonic and hypersonic turbulent boundary layers is lake of reliable and detailed data base as the reference. For hypersonic boundary, not only the drag problem, but also the heat problem is essential for engineers. While for heat problem, even the boundary condition at the wall can not be exactly specified in experiments.

However, for some simple flows, for example, boundary layers of flat plates and cones, DNS can be applied to obtain detailed flow field, which can serve as the guide for turbulence modeling. Usually, it is time consuming for doing the DNS, unless appropriate inflow condition can be specified for the DNS. In fact, we have devised a method to specify appropriate inflow condition [6], which greatly facilitated us to do DNS for supersonic and hypersonic boundary layers. And with the so obtained data base, some idea of how the turbulence models of supersonic and hypersonic turbulent boundary layers can be improved have been drawn.

Most existing turbulence models end up with the coefficient of eddy viscosity μ_T , and by applying the idea of turbulent Prandtl number Pr_T, also the coefficient of eddy heat conductivity k_T . In some very popular models, such as k-ε, k-ω, and SST models, μ_T is expressed through the turbulent kinetic energy k and the dissipation rate ε as

$$\mu_T = 0.09 f_\mu \rho k^2 / \varepsilon \qquad (1)$$

Or through k and ω as

$$\mu_T = \rho k \omega \qquad (2)$$

Then through turbulent Prandtl Pr_T number, one obtains k_T as

$$k_T = C_p \mu_T / Pr_T \qquad (3)$$

where C_p is the specific heat under constant pressure.

After we have obtained the data base from DNS, we can first check if the above expressions are correct or not.

In Fig.5a, three curves of μ_T are shown, which are for the case of a Mach 6 turbulent boundary layer on a cone with adiabatic wall, one is derived directly from the DNS data base, the other two are obtained through equations (1) and (2), and the *k*, ε, and ω are derived from the DNS data base. One can see that not only the magnitude, but also the general trend of the curves are different for those obtained directly from DNS data base and those obtained indirectly through equations (1) and (2), implying that the expression (1) and (2) are in principle incorrect. In fact, expression (1) came through dimensional analysis by Kolmogorov, which might be true for isotropic, homogeneous turbulence, but certainly can not be true for turbulent shear flows, because for the former, there is no mechanism to generate new turbulent energy, while for the latter, there is mechanism to generate turbulent energy in the wall region, which is certainly inhomogeneous and has not been considered by Kolmogorov in his dimensional analysis.

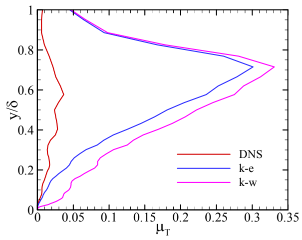


Fig.5 The distribution of μ_T versus y/δ
y the normal coordinate, δ the boundary layer thickness

The conclusion is, there is no way to make the k-ε, k-ω, and SST models rational, its accuracy depends solely in adjusting the parameters they have, and must be done case by case.

On the other hand, model proposed by Baldwin and Lomax, i.e. the BL model, is often seen as unsophisticated, though it is often favored by engineers due to its simplicity. The model divides the flow into two regions. In the wall region, the coefficient of eddy viscosity is determined by the mixing layer theory, and in the outer layer, a certain empirical function is used. The two layers match at a location where their coefficients of eddy viscosity match each other. In the literatures, the distance of the matching location from the wall is assumed to be less than 0.09δ.

However, for hypersonic boundary layer, BL model often yields result not accurate enough both for the drag coefficient and the heat transfer coefficient or temperature at the wall. From our DNS results, it is found that for hypersonic turbulent boundary layer, the distance of the matching point from the wall can be much bigger than 0.09δ, and if one express the coefficient of eddy viscosity in the outer region also in the form of mixing length theory, then the mixing length in the outer region can be virtually constant. See Fig.6, which is the result of DNS for a Mach 6 hypersonic boundary layer with given wall temperature. The ordinate is the mixing length, the abscissa is the distance from the wall, normalized by boundary layer thickness. The matching point is nearly 0.5δ. Of course, the distribution of the mixing length in the outer region may be not so regular for all the cases, but some deviation from being constant in the outer region does not have much effect on the mean flow profile, as in the outer region, the gradient of the mean velocity is very small.

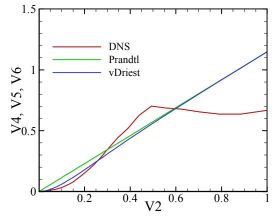


Fig.6 The distribution of the mixing length

Therefore, we can reformulate the BL model as follows: in the inner layer, the coefficient of eddy viscosity is given by Prandtl's mixing length theory (may have a certain modification as shown in the literatures), and in the outer region, the eddy viscosity is also determined in the form of mixing length theory, but the mixing length is constant. The two layers join each other at a pre-determined location, and the mixing length in the outer region can be determined by matching the coefficients of eddy viscosity there. The formulation is quite simple, but the problem is how to determine the matching location. For which we need to do DNS to find its dependence on certain parameters, such as the Mach number, Reynolds number etc. As preliminary conclusion from our DNS results, the boundary layer thickness, when measured in wall unit, decreases as the Mach number increases, while increases as the Reynolds number increases, but the distance from the wall of the matching point may be virtually constant as expressed in terms of the wall unit.

For the heat problem, there is another problem affecting the accuracy of the result, namely the assumption that the turbulent Prandtl number is constant through out the whole boundary layer. Usually, the turbulent Prandtl number is assumed to be 0.9. But from our DNS result, it is found that this is not true, especially near the wall, where its value can be appreciably bigger than 0.9. In fact, this problem has already been pointed out by several authors, and ways for its improvement has been proposed, but none of them works well for hypersonic turbulent boundary layers.

We have put forward an argument that although both the eddy viscosity and eddy conductivity originate from eddies of the turbulent flow, but the ability of transferring momentum by eddies is more efficient than transferring the heat. Thus, in the wall region, where large scale coherent structures exist, the turbulent Prandtl number there should be larger than 0.9, and the deviation may be proportional to the turbulent kinetic energy.

Thus, we proposed to modify the turbulent Prandtl number as

$$k_T = F(y)\frac{c_p}{0.9}\mu_T \qquad (4)$$

$$F(y) = \frac{1}{\max[\xi(k/k_{\max}),1]} \qquad (5)$$

Where k is the turbulent kinetic energy, ξ a parameter, and $k/k_{\max}$ can be expressed as

$$\frac{k}{k_{\max}} = 1 - \left|1 - \frac{15(1-\exp(-y^+/7))}{y^+}\right|^{1.8} \qquad (6)$$

Which is obtained by curve fitting for the distribution of the turbulent kinetic energy, which is more or less universal, and its form is shown in Fig.7

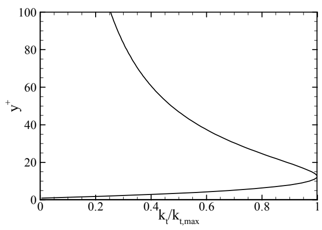


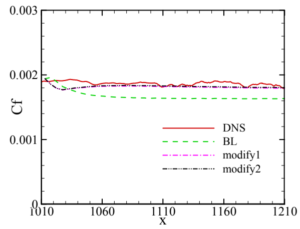
Fig.7 Distribution of turbulent kinetic energy

The parameter ξ depends on the wall temperature as

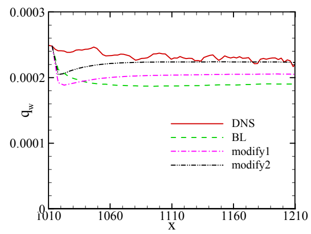
$$\xi = \frac{T_0 - T_w}{b} + 1 \qquad (7)$$

Where T_w is the wall temperature, and T_0 is the total temperature at the edge of the boundary layer. In this formulation, there is only one adjustable parameter, namely b.

Fig.8 shows the result of the modified BL model for a Mach 6 turbulent boundary layer on a cone with a cold wall. In which curve labeled DNS implies result from DNS, curve labeled BL implies result from original BL model, curve labeled modify1 implies result from the improved BL model but without the modification on the turbulent Prandtl number, and curve labeled modify2 implies the modification also includes modification on turbulent Prandtl number. Because the initial condition for the turbulence modeling is not perfect, there is a transient section for the turbulence to reach a fully develop state. So the meaningful comparison should be in the interval 1670 < x > 1210. Fig.8(a) is for the surface friction coefficient, Fig8(b) is for the wall heat flux coefficient, Fig8(c) is for the mean velocity profile, and Fig8(d) is for the mean temperature profile. We can see that the proposed improvements do work for both the drag and heat problem.



(a)



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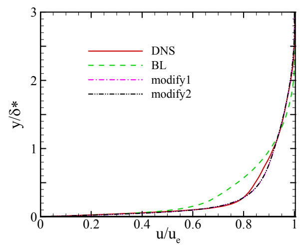

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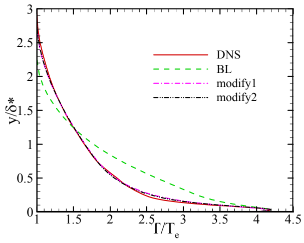

(d)

Fig.8 Results showing the improvements

For more detail see [7,8]

4. Conclusions

(1) The e^N method can works well for the transition prediction of supersonic and hypersonic boundary layers, but one should start the integration of the amplification factor not from the ZARF as in the conventional way, but from a proper location determined by receptivity consideration, and the N factor is not given by experience, but determined by the ratio of the disturbance amplitude at transition, for example 0.01~0.02 of the velocity at the edge of the boundary layer, and the guessed initial amplitude.

(2) For the turbulence modeling of supersonic and hypersonic turbulent boundary layers, it seems there is no way to make the popular k-ε, k-ω, and SST models more rational, unless more parameters are introduced. But for the simple model such as the BL model, it is possible to make further improvements for its ability to predict the drag coefficient and the heat conductivity coefficient, or the temperature, at the wall.

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