Proceedings of the 13th Asian Congress of Fluid Mechanics
17-21 December 2010, Dhaka, Bangladesh

Fluid Dynamics on Swimming Animals

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Abstract

Fluid dynamics related to thrust of swimming animals was discussed. This was based on the law of action and reaction because a force given by swimming animals to a fluid causes a reacting force from the fluid to the animals, namely thrust. The shape and the motion of the fin of fish were focused on for a thrust generation. Though the motion of living things is unsteady anytime, thrust has been estimated indirectly by using drag at steady state. Through observation of unsteady motion of a jellyfish and fish, we found that generation of a vortex ring relates to the thrust. The vortex ring is common to many cases of propulsion of small swimming animals. Therefore, dynamics of a vortex ring may give a way to evaluate thrust directly. A vortex ring generated by an accelerating disk was investigated experimentally for confirming the relation between the producing force and a vortex ring. According to morphological consideration about the shape of fins to produce thrust, a convenient way to estimate thrust of small animals directly based on the vortex dynamics was proposed.

Keywords Fish, Tail fin, Motion analysis, Unsteadiness, Added mass, Vortex ring, Circulation

1. Introduction

How does fish get thrust? This question is a main topic of my presentation. The equation of its motion is usually expressed as $ma=T-D$. Here, m is its inherent mass, a is an acceleration of motion, T is thrust, and D is drag acting on its body. When we observe its motion, we can obtain its acceleration by a motion analysis of the recorded movie. The drag D shows usually shape drag, which is expressed by the drag coefficient based on frontal area of a body. This is adequate for known blunt shapes, e.g., a circular cylinder, a sphere, etc. For stream-lined shapes the drag is mainly due to friction. We need to know about boundary layer flows along a body surface in this case. However, this is not easy because of a measurement of velocity profiles. Therefore, we have estimated the thrust for the stream-lined body as summation of measured *ma* and D which is calculated by a friction coefficient C_f of a boundary layer. As the friction drag is enough small for the stream-lined body in general, we occasionally neglect it, and then we make *ma* thrust. This is not far from truth for flows at high Reynolds number, but this is inadequate for unsteadily swimming animals at low Reynolds number.

Does the text book on fluid dynamics give sufficient knowledge for the drag coefficient of unsteady motion? The answer is "no". There has not been the theory which gives drag coefficients of a body in unsteady flow except for concept of an added mass. Moreover, we have not understood the friction enough yet. Of cause, the friction coefficient of a simple flat plate in laminar and turbulent boundary layers is well known. However, we do not answer for the friction coefficient of animal skins: hairy skin, slick skin, slimy skin, scaly skin, etc. Living things ought to invent suppression of friction drag for survival. One of them is a shark skin to suppress bursting events in a turbulent boundary layer. The other is a slimy skin which is formed by secretion of mucous material on the surface of fish. Fish and eel secrete mucous. However, we have not known its effects on suppression of friction drag yet. In the case of polymer solutions, Tom's effect is well known as an enormous decrease of friction drag. This is explained by suppression of turbulence in a turbulent boundary layer. In the case of the mucous skin, properties of a fluid, that is, water, is not changed by the skin. Thus, if a suppression of friction drag is recognized, it may not be the Tom's effect. We should understand molecular interactions between the material of the surface and fluid.

The way to get thrust is various, but obtaining mechanism is rather simple than that of drag. The mechanism is principally based on reaction of motion. Utilization of flow yields various motions of items like fins or arms as shown in Fig. 1. One obtains thrust by a reaction based on drag in a low Reynolds number, Re, whereas one utilizes lift in a high Re. Here, you should brand on your mind that the drag in this case is not drag acting on a body expressed in the equation of motion mentioned above. This is the drag of the item to produce thrust, namely, the reaction force against motion of the item. Small animals like as plankton use the drag without exception. Items are a flagellum, cilia or feelers for plankton, and fins for fish, legs or arms for mammals which are not good at swimming, e.g., human. Their motion except for fins is a reciprocating which consists of power stroke and recovery stroke. It needs difference in the power and recovery strokes to obtain a net thrust. The Reynolds number of the living things which use such the reciprocating motion is much smaller than that of fish except for human. Roughly speaking, if Re is smaller than 10^{-3}, one propels using flagella or cilia. As that of fish is 10^2~10^5, its propulsion is produced by a fin. The motion of fin is also reciprocating, but there is no difference of stroke. Namely, the motion of fin in the one direction is the same as the other direction as you can picture to yourself. How does it produce thrust? This is my major question in this presentation. On the other hand, the item is a fin for sharks, dolphins or whales, and arms for turtles in the case of higher Re than 10^5. The shape of a cross section of the fin or the arms is a symmetrical wing section. This is a principal difference from the fin in the case of fish moving in the low Re environment. The motion of wing-like fin seems to be the same as that mentioned previously, but it is not the simple reciprocating motion. The motion consists of heaving and pitching as to produce lift force. It is the so-called a flapping, but it is not the same as the one of birds because of lack of leading-lugging motion.

Fish has a pair of pectoral fins, a pair of ventral fins, an anal fin, a tail fin, and a dorsal fin as seen in Fig. 2. A pair of the pectoral fins is supposed to be used to keep an upright posture. As seen in the figure of a cross section of fish, straight posture is unstable because the center of buoyancy is below the center of gravity. The pectoral fin yields the moment of a force around the center of gravity to keep the balance. Therefore, the pectoral fin is necessary to be the drag-based shape for the sake of a quick response (Drucker and Lauder [1]). Some fish uses the pectoral fin also for both back and forward moving. The dorsal fin is to stabilize the body against rolling and assist in sudden turns. It is the usage on the analogy of a vertical stabilizer of an airplane. The tail fin is used to generate thrust as discussing later. Other fins are not clear about their usage. The silver shark which is a kind of fresh water tropical fish has large fins as seen in Fig. 2. The shape of the dorsal fin is similar to that of a shark. It is not clear whether the usage is the same or not, because the Re number is extremely different. The tail fin consists of two knives. This shape is similar to that of the shark, but is different mechanism of the thrust generation as discussing later.

The ways to get thrust by means of the tail fin are divided into two classes: one is drag-based in a low Re, and the other is lift-based in a high Re as shown in Fig. 3. We can also classify the shape of fin using the drag-based way into two types: one is an oval or a trapezoidal fin and the other is a double acute-angled triangle, that is, a concave polygon. The former is fit to use an abruptly accelerating motion. When the fish catches prey or avoids predators, it needs to produce a large force in a moment. It works well to push an added mass of fluid. Therefore, fish with such the tail fin utilizes the impulse due to the potential flow. On the other hand, the latter is fit to produce a drag-based force in a steady swimming. The force is related to low pressure due to a vortex along perimeter of the fin, so that augmentation of peripheral length of the concave polygon is profitable to produce a large force. Each acute-angled triangle can move separately. I guess that it is for subtle control of force. I think that a vertex at steep concave is important because of its singularity for severing relations with neighboring vortices.

The motion of swimming animals expressed by *ma* is yielded by a result of *T*-*D* as mentioned previously. Thrust *T* contains not only the force produced by animals themselves but also gravity and buoyancy forces in water. Drag *D* also contains them. Because of definition the thrust is a resultant force in the direction of movement and the drag is that in the opposite direction of movement. Therefore, we need to take account of gravity and buoyancy when we consider the observed motion. I focus on just fluid dynamics related to thrust generation mechanisms.

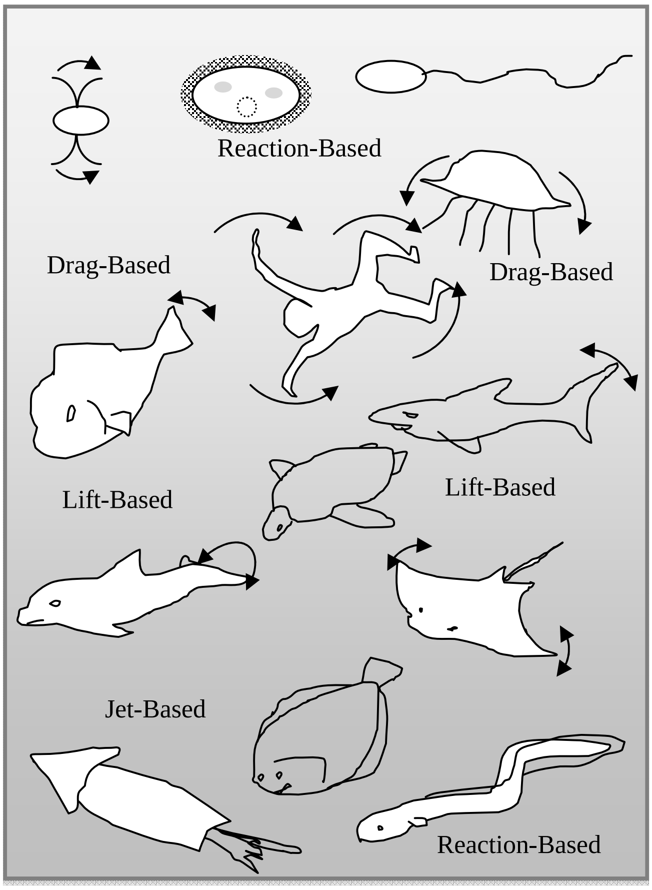
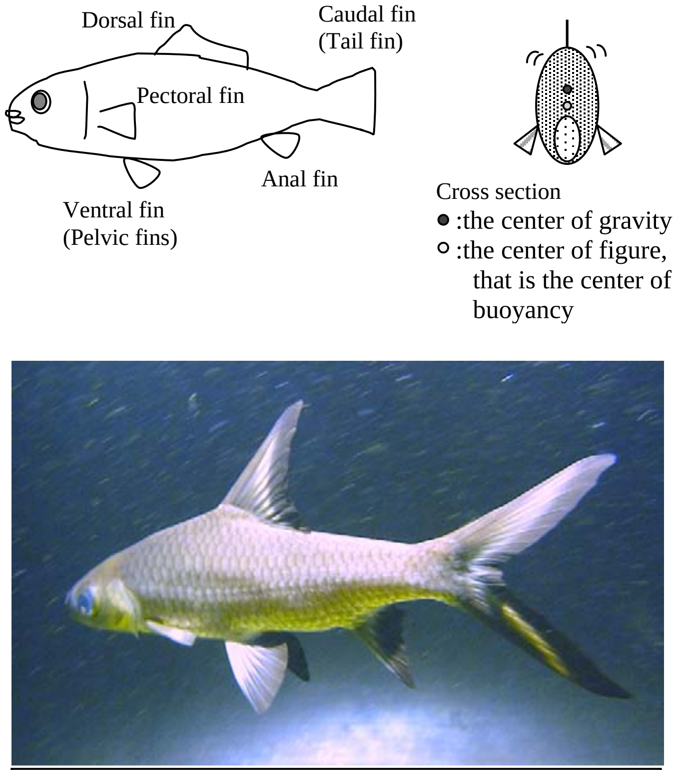


Fig. 1 Schematic drawing of different swimming animals classified by thrust acquirement

In this paper, I will show a way to estimate thrust by a vortex ring as a representative flow element which is pr



Bala Shark (Balantiocheilos melamopterus) is called "silver shark" because of its torpedo-shaped body and large fins.

Fig. 2 Fins of fish

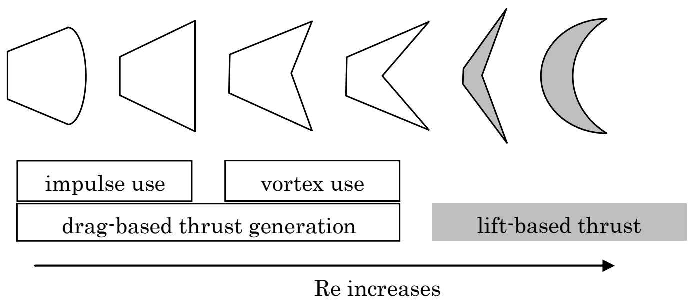


Fig. 3 Various shape of tail fin and way to produce thrust to the Reynolds number

introduction I show the equation of motion and relation between the thrust term and motions of items, and the mechanisms to produce thrust based on each drag and lift by means of a fin in Section 2. The experimental results on vortex ring produced by accelerating disk and a way to estimate thrust through observations of motion of a fin and shape of fin are shown in Section 3. I will show a way to evaluate thrust directly related to generation of the vortex ring by a fin according to morphological consideration about shape of fins to produce thrust. A consideration of drag-based thrust is presented in Section 4 and life-based thrust is presented in Section 5. Finally, conclusions are given in Section 6.

2. Expression of thrust in equation of motion

An unsteady motion of fish is expressed as follows.

$$m\frac{du}{dx} \quad (1)$$

Here, an acceleration of body *a* is expressed by du/dt in Eq. (1). The notation *m*' seen in the third term in left side shows the added mass due to unsteady motion. Namely, fish have to generate additional force to move water of *m*' when it moves unsteadily. This is the difference from estimation of thrust force in the steady state. Thrust is expressed by *T*, and drag expressed by *D* which contains shape drag and friction drag in Eq. (1). The drag is written by dynamic pressure as follows.

(2)

Here, C_D is a drag coefficient when the area *A* is taken by a wetted surface area of fish not a projected (frontal) area. Thus, the drag contains both shape and friction drag. The value of C_D of fish is 0.004~0.015 referred from the textbook written by Vorgel [2]. Since fish has a streamlined shape, the shape drag is considered small. Thus, the value is regarded as the friction drag. However, most of these values were obtained by measurements in a steady flow. The skin friction of fish is summarized in the book written by Azuma [3]. The value of C_f which is the drag coefficient based on the wetted surface area is in a range from 0.003 of a porpoise to 0.5 of a box fish. These values are larger than that of a flat plate with the same surface area in the laminar boundary layer. Anyway, the drag has been usually estimated by Eq. (2) assuming the drag coefficient. From motion *ma* measured by a motion capture system thrust in a steady flow is estimated as summation of *ma* and the drag. Moreover, if fish swims at constant speed, magnitude of thrust is equal to that of drag because of *ma*=0 due to *a*=0. In this case, the observation by the motion analysis gives only the speed but the force. Namely, we need further information about drag coefficient and surface area to estimate the drag. This means that the accuracy of estimated thrust depends on that of drag even if we observe the motion with a constant speed as the simplest one. Of cause, there is the way to measure the drag directly in a uniform flow with a constant speed. Usually fish or its model whose shape is copied a real one is tethered in a water channel. Mostly scale and properties on skin are not copied to the model. Therefore, the drag obtained in this kind of experiments is the shape drag of the 'model' including a little friction. According to data summarized in the book [3] the drag coefficient of a streamlined body of revolution depends on fineness ratio which is the ratio of the body length to the body diameter. The drag coefficient based on the frontal area attains the minimum at the ratio=2.5, whereas that based on the wetted surface area seems to be inversely proportional to the ratio. We have not understood the drag of even the basically simple shape of the body yet. Nevertheless, researchers have used Eq. (1) to estimate thrust through observation of freely swimming fish. This is based on the frozen flow hypothesis in a short time, namely, a quasi-steady treatment.

On the other hand, thrust T has been considered to be hard to estimate directly because it depends on the way to generate thrust, e.g. fin, arms, legs, etc. There have been no expressions of T like as that of drag. This is a major reason why thrust has been indirectly estimated from Eq. (2) by using a given drag coefficient of steady states as mentioned above. If we can estimate thrust directly from a motion analysis, it contributes estimation of thrust of microbe and plankton, and also development of biomechanical engineering to make a biomimetic fish robot. For estimating thrust directly we need to understand fluid dynamics to give thrust as a reacting force to the body. Motion of a fin may give momentum to a fluid, so that a related flow arises. If we know the rate of change of momentum of a fluid system, we can catch the force given to it by the fin according to the Reynolds transport theorem. This is a basic idea to understand the thrust given by a fin. The basic idea is not new but classical one which was developed by Lighthill [4]. This is based on a force analysis for an undulatory motion of the body. Fin was dealt as an extended part of the body, but its specific characteristic on thrust generation did not consider. The vortex structures generated by a body undulation are shed behind the fish. Since the vortex street seems to be a jet-like pattern, generally it has been considered that fish gets thrust by the jet. I doubt the mechanism of obtaining thrust by the jet because the vortices beside the body should be drag on the body due to separation of the boundary layer on the body surface and jet-like flow behind the fish is a result of induced velocity field by shed vortices. When fish gives a fluid momentum, the fluid must gives the fish momentum according to the law of action and reaction. If a mass of the fluid received momentum from the fish is extraordinarily large, fish moves but the fluid does not. In this case, there is no flow. However, vortices are observed as a result of momentum exchange between fish and a fluid. Therefore, a mass of fluid is regarded as that of the vortex, and change in velocity of the fluid is regarded as that of the vortex. Then, if the motion and volume of the vortex are measured, the force given by fish can be known as a change in momentum of the vortex. This is the basic concept to estimate thrust in this paper.

When we observe a flow induced by a reciprocating fin, we can see vortex rings as characteristic elements of the flow. Since the vortex ring is a production as a result of motion of a fin, if we can get the momentum change of the vortex ring, we can understand the given force as mentioned above. This will be described detail in Section 3. Basically, motions of living things are unsteady. The vortex ring is one just generated in a short term during an accelerated or decelerated stroke of the reciprocating motion of the fin. Therefore, the potential theory is applicable to the flow including both the vortex ring and the fin. This makes the relation between the vortex ring and a quick motion of the fin simple because we do not need to take account of viscosity effects on a result. It becomes possible to consider unsteadiness according to a time derivative of a velocity potential. In other word, the unsteadiness of the velocity potential gives an origin of the added mass. Therefore, the estimation of thrust based on unsteady generation of a vortex ring is less uncertainty than that based on drag described previously. Though the item to produce thrust is different as mentioned previously, the manner to produce thrust is governed by the same physics.

3. A starting vortex ring

When we observed a jellyfish swimming [5], we had doubts about usual understanding of its thrust based on momentum of a jet ejected by contraction of its skirt. We found a vortex ring instead of the jet when the jellyfish accelerated. The vortex ring was generated at the edge of the skirt by its quick motion, that is, a snappy motion. We have not had enough information about a relation between a vortex ring generated by a body started impulsively and working force for its generation. We conjectured that a vortex ring may arise in accordance with an abrupt motion of a thin disk and its energy should be given by work done by it. Thus, we studied a starting vortex ring generated behind an accelerated circular disk as a model of an impulsive starting body. We adopted gravitational force as known one to give accelerating motion, so that we can determine rigorously the motion of the disk. Since the energy of a steady vortex ring with a thin core is presented as a function of its dimensions and circulation by Saffman [6], we tried to apply it to a developing vortex ring accompanying with the accelerated circular disk to know the relation between change in energy of the vortex ring and work done by the disk. We measured change in velocity-field around the accelerated circular disk by a PIV system, and obtained the relation between the change in circulation of the vortex ring and force given by the disk. After checking the momentum transfer, we applied it to the estimation of thrust by an accelerated non-circular disk that has the same area as the circular one.

We carried out experiments in a water tank with 400mm in depth, 300mmin width and 300mm in length. The experimental setup is shown in Fig. 4. A circular disk with 50mm in diameter, L, and 3mm in thickness moved impulsively from a stationary to a dynamic state along the center of the water tank by means of gravity. The loaded weight was changed from 0.1N to 1.0N. We measured flow fields around a circular disk by using a PIV method. The flow field was illuminated by a light sheet and taken by a high-speed CMOS camera using 2200 frames per second (fps) for velocity measurements. We used polystyrene particles (ORGASOL) with 50μm diameter as tracer particles. The circulation of a starting vortex ring was calculated at the same non-dimensional time, $t^*=0.5$. Here, t^* is defined as follows.

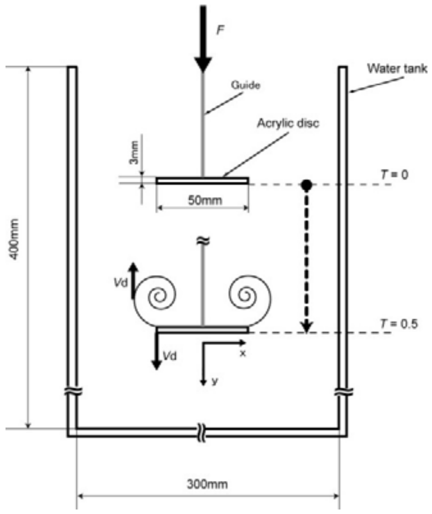


Fig. 4 Coordinates and experimental setup for a starting vortex ring

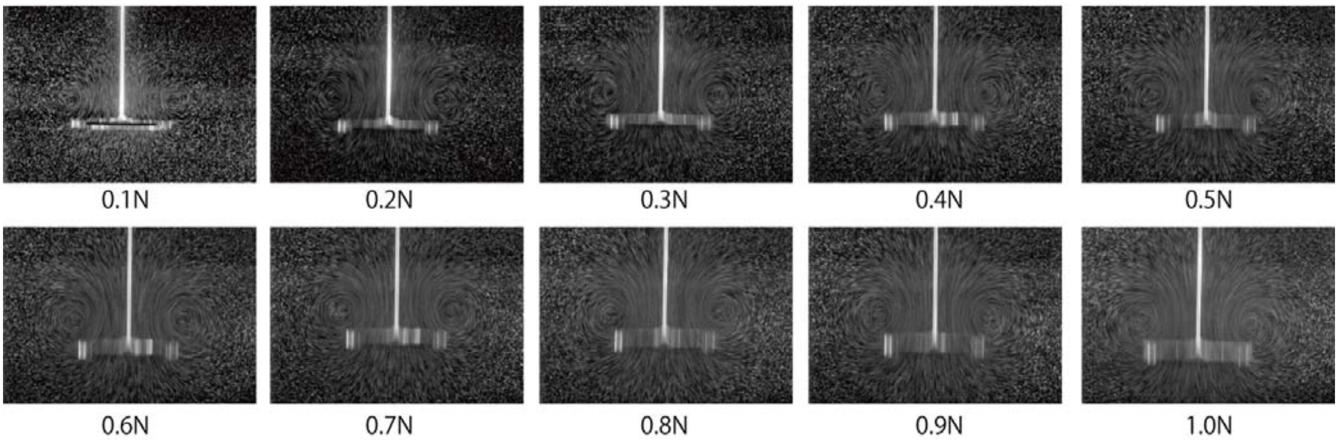


Fig. 5 Path lines visualized around the disk moved by forces in a range from 0.1N to 1.0N

$$t^* = \frac{kV_{dt}}{m + m_l + m'}t \quad (3)$$

Here, V_{dt} is a terminal speed of the disk, m is a mass of the disk, m_l is a mass of a load, m' is a virtual mass which is defined by Eq. (4), t is a measured time, and k is a drag coefficient which is defined by Eq.(5).

The added mass of the circular disk moving in the vertical direction against its face is expressed as follows.

$$m' = \frac{8}{3}\rho L^3 \quad (4)$$

The arranged coefficient of drag is defined as follows.

$$k = C_D \frac{1}{2}\rho A \quad (5)$$

Speed u of the disk is obtained as a function of time t by solving an equation of motion of the disk falling in water by gravity.

$$u = \sqrt{\frac{Mg}{k}}\tanh\left(\sqrt{\frac{kg}{M}}t\right) \quad (6)$$

Here, M is a summation of m, m' and m_l. Adjusting k we can make the observed change in speed of the disk coincident with the theoretical change given by Eq. (6). Namely, if thrust is known, we can obtain the drag coefficient given by Eq. (5) according to this experiment. Anyway, now we have a known accelerating motion. Then, let's observe a vortex ring generated by this motion.

The flow field presented by path lines around the disk observed at $t^*=0.5$ is shown in Fig. 5 for cases of 0.1N to 1.0N loads. These pictures were obtained from movie taken by 60 fps. The curved path lines behind the disk shows a flow pattern in a section through the centerline of a vortex ring. The center of rotating motion presented by these path lines shows that of the vortex core. The position and diameter of the vortex core seems to be independent of difference of load. The velocity profile in the section through the centerline of the vortex ring is shown in Fig. 6 for each case. The vertical axis shows change in magnitude of velocity component in the vertical direction. The horizontal axis shows non-dimensional distance, l/L. Here, l is distance measured from the center of the vortex ring. The positions at which velocity attains the maximum and minimum values show the edge of the vortex core. The center between these positions is that of the vortex core, and the length between them is the diameter $2r$ of the vortex core. The absolute values of the maximum and minimum values are the same, because these show the circumferential velocity at the radius of the vortex core. The length measured from the center of the vortex ring to the center of the vortex core shows the radius, R, of the vortex ring. The R coincides with the radius of the disk, being independent of load. The velocity, v_s , at the edge of the vortex core is plotted against the speed, V_{dm} , of the disk at t*=0.5 in Fig. 7 for each load. The velocity at the edge of the vortex core, that is, absolute value of the maximum velocity, correlates with the speed of the disk. This means that the circumferential velocity at the edge of the vortex core can be expressed by the speed of the moving plate generating a vortex ring.

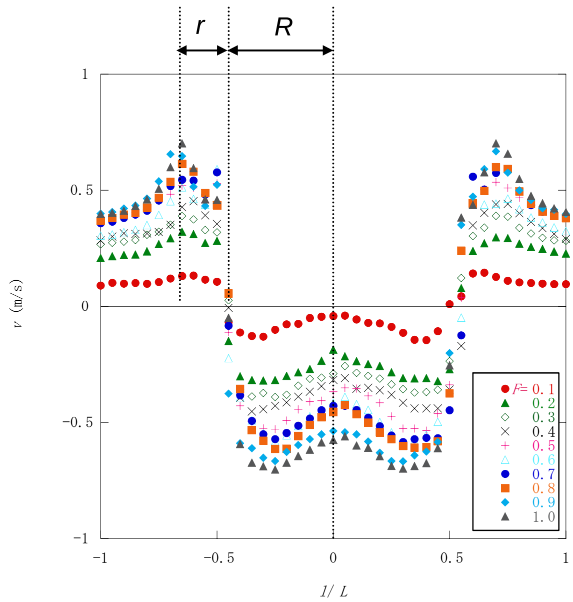


Fig. 6 Velocity profiles of vortex ring at t*=0.5

The center position and radius of the vortex core are shown in Fig. 8. The vertical axis shows non-dimensional length, y/L, in the vertical direction. The horizontal axis shows non-dimensional length, x/L, in the horizontal direction. Here, y is a length measured from the disk model in the vertical direction, and x is a length measured from the center of the disk in the horizontal direction. The gray bar is a position of the disk. The relative position to the disk of the vortex core obtained for different loads is the same as mentioned above. The time change in circulation of the vortex ring and that in speed of the disk

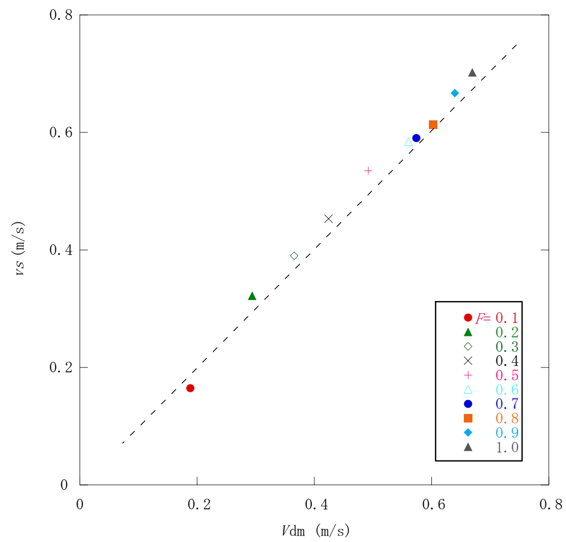


Fig. 7 Relation between V_{dm} and v_s at t^* = 0.5

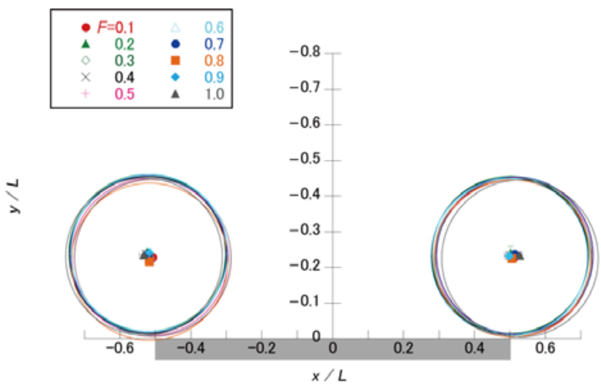


Fig. 8 Comparison of center position and radius of core of vortex rings generated by the disk moved forces from 0.1N to 1.0N at t*=0.5

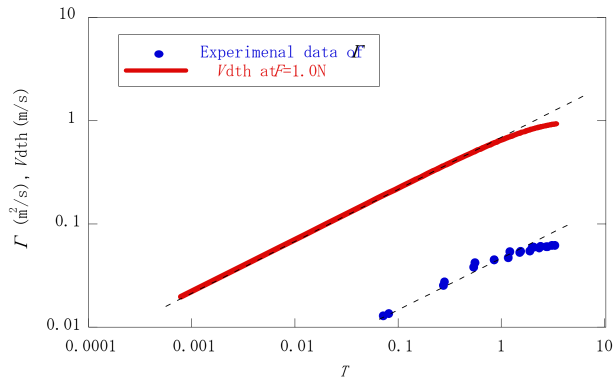


Fig. 9 Time history of circulati t^* speed of the disk in the case of load of 1.0N

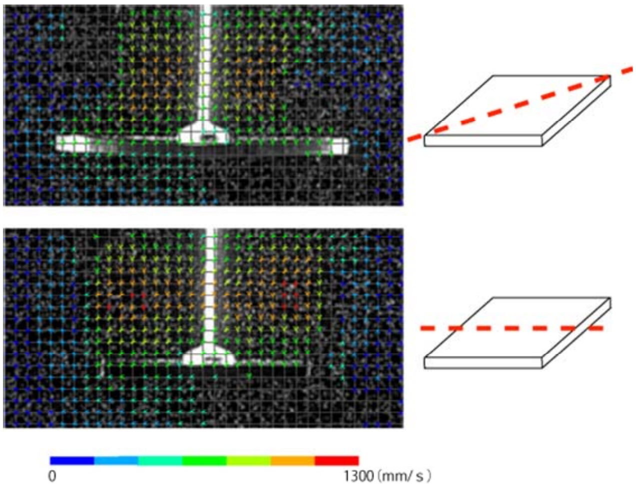


Fig. 10 Vector field around the square disk at 1.0N

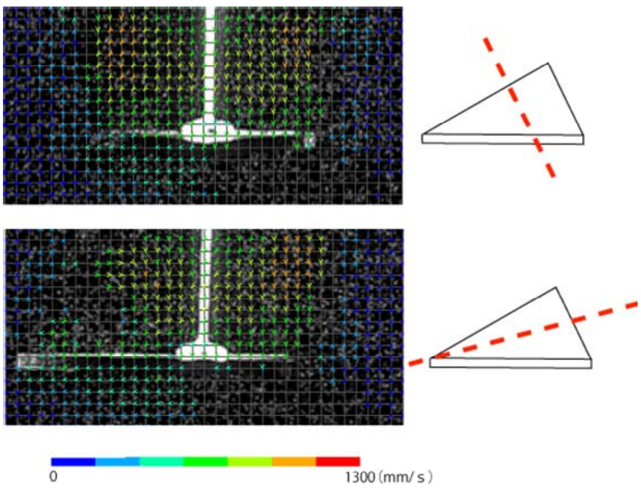


Fig. 11   Vector field around the triangle disk at 1.0N

at given load of 1.0N is shown in Fig. 9. The vertical axis shows circulation and speed of the disk. The horizontal axis shows non-dimensional time *t**. The blue plot shows circulation. The led line shows speed of the disk. Both the circulation of the vortex ring and velocity of the disk increases with time. The ascendant tendency of both quantities coincides with each other. Namely, the circulation of the vortex ring correlates to speed of the disk. Summarizing these results on a vortex ring generated by the accelerating disk, the size of the vortex ring is determined by that of the disk, and the velocity at the vortex core is determined by the speed of the disk.

The energy E of a steady vortex ring with thin core is expressed as follows [6].

$$E = \frac{\rho\Gamma^2 R}{2}\left(\log\frac{8R}{r} - \frac{7}{4}\right) \qquad (7)$$

Here, ρ is density of water. If the size parameters of a vortex ring are expressed by κ, Eq. (7) becomes simple as follows.

$$E = \kappa\Gamma^2 \qquad (8)$$

Here, κ is equal to $\rho R/2\{(\log 8R/r)-7/4\}$. On the other hand, work done by the disk is given by the following Eq. (9).

$$W = FV_{dm} \qquad (9)$$

Here, F is force acting on a fluid through the disk. Since the work done by the disk is equal to time change in energy of the vortex ring, the time derivative of Eq. (8) is equal to Eq. (9). Then, we can obtain the following relation.

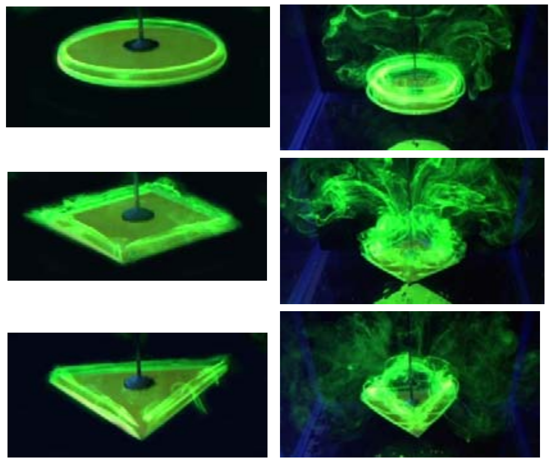


Fig. 12 Starting vortex for different shape of disk at a) t^*=0.01 and b) t^*=0.5

$$\frac{dE}{dt}$$

Substituting the circulation Γ expressed by $\Gamma=2\pi r v_s=2\pi r V_{dm}$ into Eq. (10), we can obtain the following expression.

$$F = \frac{1}{V_{dm}}2\kappa\Gamma\frac{d\Gamma}{dt} = 4\pi r\kappa\frac{dV_{dm}}{dt} \qquad (11)$$

Equation (11) shows that the force acting on fluid is in proportion to the product of κ, r and the acceleration of the disk. Therefore, if we obtain the size of a vortex ring and an acceleration of the disk which gives force, we can calculate the force acting on fluid according to Eq. (11). Since the thrust is equal to the reaction force from fluid, the force given by Eq. (11) is just the thrust generated by the disk. We know the size of the vortex ring already as mentioned previously. To estimate thrust we may observe just the accelerating motion of the disk.

4. Drag-based Thrust

How to do in the case of a non-circular disk? To check this we examined a non-circular disk with same area as a circular disk. We observed its motion by a motion-capture system and measured flow fields around it by the same PIV measurement system as that of the circular disk. A square disk and an equilateral triangle disk were used as a non-circular disk. The square disk was 44.3mm in one side length, and 3mm in thickness. The equilateral triangle disk was 67.3mm in one side length, and 3mm in thickness.

The velocity vector fields around the square disk with the same area as the circler disk are shown in Fig. 10. The upper figure shows a velocity vector field in a cross section through diagonal line of the square disk as shown in a right illustration. The lower figure shows that in a mid-section of the square disk. We can recognize the vortex-like flow at sides but cannot observe this flow pattern at corners of the square disk. On the other hand, the velocity vector fields around the equilateral triangle disk are shown in Fig. 11. The upper figure shows it in a cross section through diagonal line of the equilateral triangle disk as shown in right illustration. The lower figure shows that in a mid-section through a vertex angle of the equilateral triangle disk. We can recognize the vortex-like flow at sides but cannot see the flow at a vertex angle. A thin vortex adheres at the edge of the non-circular disk according to dye visualization of a newborn vortex as seen in Fig. 12. The length of a line vortex coincides with that of a side of the disk just after starting to move. Since vertices are singularities, line vortices at each side cannot connect each other at first. Feet of each line-vortex must attach at each neighbor vertex. When we looked at the flow field in a section through a vertex, we never recognized the vortex-like flow because of the above reason. Reconnection of vortices occurs when the vortex core becomes fat later. Then, the line vortices form a vortex ring as seen in Fig. 12b).

We tried to convert the line vortices into an equivalent vortex ring as follows. The radius of the equivalent vortex ring is given by Eq. (12).

$$R = \frac{L_{cd}}{2\pi} \tag{12}$$

Here, L_{cd} is the perimeter of the non-circler disk. From measurement of the starting vortex shown in Fig. 12a), we obtained $r=R/20$. Since this satisfies the condition of thin vortex core, and then the size parameter κ is calculated as $\kappa = 0.26rL_{cd}$ according to the definition. Recalling the circulation Γ is $\Gamma=2\pi r v_e$, time change in Γ is expressed as follows.

$$\frac{d\Gamma}{dt} = 2\pi\left(\frac{R}{20}\right)\frac{dv_e}{dt} = 2\pi\left(\frac{1}{20}\frac{L_{cd}}{2\pi}\right)\frac{dv_e}{dt} = \frac{L_{cd}}{20}\frac{dv_e}{dt} \tag{13}$$

Substituting κ and Eq. (12) into Eq. (10), finally the thrust by a non-circular disk can be obtained if the perimeter and the acceleration of the non-circular disk are measured.

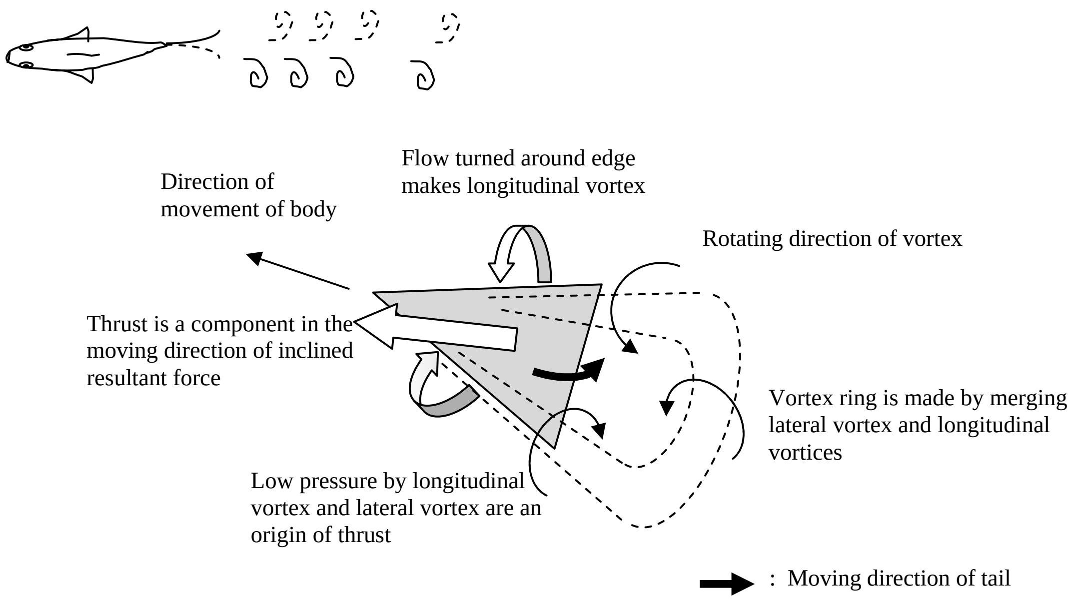


Fig. 13 Vortex ring by tail fin

$$T = \frac{1}{v_e} 2\kappa\Gamma \frac{d\Gamma}{dt} = 4\pi r 0.26 \rho L_{cd} \left(\frac{L_{cd}}{20} \frac{dv_e}{dt} \right) = 2.6 \times 10^{-3} \rho L_{cd}{}^3 \frac{dv_e}{dt} \quad (14)$$

The magnitude of thrust is proportional to the cubic of the perimeter. If the area of the disks is the same, the perimeter of the disk is 6.28*R* for the circle with the radius *R*, 7.10*R* for the square and 8.08*R* for the equilateral triangle. The perimeter of the equilateral triangle disk is 1.3 times longer than that of the circular disk. Therefore, from Eq. (14) the thrust generated by the equilateral triangle disk is twice that of the circular disk.

The thrust given by Eq. (14) is the force in the direction of movement of the disk. A frog uses it as a paddling for its swimming. It is convenient to estimate thrust because we only measure the perimeter of a webfoot of frog and acceleration of its motion. When we apply it to the tail fin of fish, we need to additionally measure change in angle of the face of the wagging tail fin to take a component of the force calculated by Eq. (14) in the direction of the moving body. The speed of the tip of the tail fin is a representative speed as v_e, which is calculated from the angular velocity and the longitudinal length of the tail fin. Therefore, if we obtain the wagging motion of the tail fin, we can estimate thrust.

As known from the result shown in Fig. 12, line vortices at sides and a lateral vortex at the tip of the tail fin will be generated by the wagging tail fin. These vortices connect with each other later, and then a vortex ring is made as shown in Fig. 13. Low pressure by these vortices is an origin of thrust as described above. The longitudinal parts of the vortex ring originated from the vortices at sides are stretched due to the flow in the backward direction to motion of the body. The lateral vortex is swept away by the negative flow. Since the longitudinal vortices become thin by stretching, the rotating velocity of the vortices becomes fast according to the law of conservation of angular momentum. This brings forward dissipation of their vorticity, and then the rotating velocity is reduced. Therefore, their rotating flows are hard to see experimentally. On the other hand, the lateral vortex becomes stronger by vorticity supplied from the separated shear layer from the tip of the tail fin. At the turning point of wagging, the strong lateral vortex is shed from the tip due to inertia. In general, we can see two lines of the shed vortices behind fish as seen in an illustration in Fig. 13. Since these seem to be similar to the flow in a plane jet, ones have misunderstood as jet propulsion. These vortices are the result of impulse given by the wagging fin. Fish produces vortices but does not produce jet.

Swimming modes for locomotion of fish were widely reviewed by Sfakiotakis, et.al. [7]. Two mechanisms of drag-based thrust generation were introduced: one is "vortex peg" mechanism, and the other is "undulating pump". The former was regarded as an attached vortices to the body. It was considered that the fish transform the rotational energy of the vortices to the locomotive energy. The latter was to create a circulating flow around the inflection points of the body. The propagating circulating-flow was considered to merge with the bound vortices created by the tail and shed in the wake. These suggest that the vortices relate to the thrust generation. However, I think that the vortices do not generate thrust but thrust generates vortices.

5. Lift-based Thrust

A whale propels by means of the tail fin moved up and down as like as an orca and a dolphin. A shark also swims by means of the tail fin wagged from side to side. These reciprocating motions resemble that of small fish mentioned previously, but the mechanism of producing thrust is different in principle. This difference comes from the size and mass of the body. The Reynolds number for cetaceans is in a range from 10^6 to 10^8, whereas that for various fish is in a range from 10^3 to 10^6. The huge muscular power is necessary for the large swimming animals to move the tail fin like the small fish if they use the drag-based propelling power. Therefore, they take advantage of lift generated by a wing. The direction of fin's motion of small fish coincides with that of the force pushing a fluid, whereas that of the large swimming animals is perpendicular to that of thrust because of usage of lift. The shape of the wing is lunate in common to not only cetacean but also shark and sailfish. The large aspect ratio of the lunate tail is of advantage to reduction of the induced drag and the moment of inertia. Its tapered swept-back shape is superior in separation due to augmentation of the angle of attack along the wingspan from root to tip. To generate lift effectively during the reciprocating motion, the shape of the wing section needs to be symmetric to the cord-axis as shown in Fig. 14. The reciprocating motion consists of heaving and pitching. The heaving is the motion in the transverse direction, and pitching is the rotating motion around a center of force located at a quarter of the cord length from the leading edge. If the wing moves symmetrically, the component of inclined lift to the moving direction of the body always becomes thrust. This motion of the wing is different from that of a bird.

The thrust *T* shown in Fig. 13 is expressed by the following equation.

$$T = R \sin\theta = \sqrt{L^2 + D^2} \sin(\gamma - \varphi) \quad (15)$$

Here, γ is an angle of a velocity triangle consisting of the heaving speed *v* and the moving speed *U* of the body, that is, $\gamma = \tan^{-1}\frac{v}{U}$. The angle φ is the ratio of drag and lift forces, $\varphi = \tan^{-1}\frac{D}{L}$. If the ratio *D*/*L* is small, the wing has a good performance, and the resultant force *R* of drag and lift is regarded as the lift.

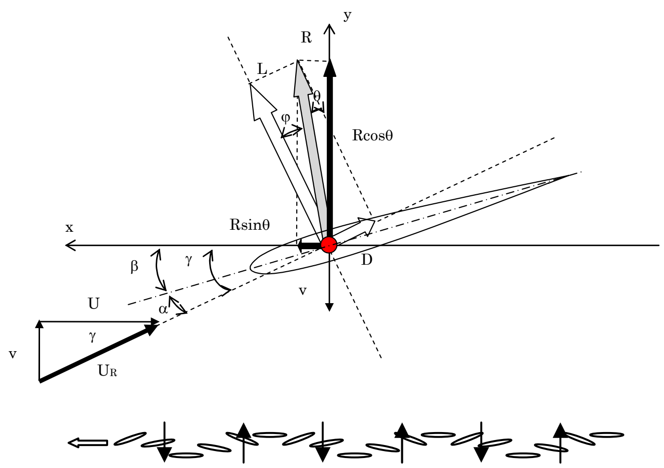


Fig. 14 Forces generated by a symmetric wing in reciprocating motion

To obtain higher thrust, from Eq. (15), the wing should generate large R, large γ and small φ. Namely, if the wing has an excellent performance, D/L becomes smaller, L is larger, and then R becomes larger. The angle φ is small if the wing has low D/L. If the speed of heaving is higher, the angle γ becomes larger. Therefore, the angle θ becomes larger for these conditions. As lift is proportional to the angle of attack α, the pitching angle β should be small, namely, it had better keep horizontal to the direction of the moving body. However, if α exceeds 15 degrees, the flow passing the wing may separate, say, stall occurs. When it occurs, lift decrease suddenly. The angle of attack depends on U and v. Therefore, β should be kept the following relation.

$$\beta \geq \gamma - 15^\circ = \tan^{-1}\frac{v}{U} - 15^\circ$$

When the heaving speed is fast, the pitching angle should be large. This relation is similar to skewness of propeller. Therefore, the large swimming animal seems to be propelled by a screw propeller.

6. Summary

Problems for estimating thrust generated by swimming animals were indicated in this paper. The generation mechanism of thrust were classified and discussed in the cases of drag-based and lift-based tail. The relation between thrust and a vortex ring was investigated by using different shaped disks which was a model of the tail fin of fish. It was confirmed that the work done by the disk was equal to the time change in energy of the vortex ring. This gave the convenient way to estimate the thrust of an accelerated disk. Namely, the time change in circulation and the size parameter of a vortex ring generated by acceleration of a circular disk give thrust according to Eq. (11). In the case of a non-circular disk like the tail fin of fish, only the acceleration and the perimeter of the non-circler disk give thrust according to Eq. (14). Therefore, when we want to know the drag-based thrust of swimming animals, we only have to observe the motion and shape of the item like a fin. On the other hand, according to Eq. (15), the lift-based thrust is obtained by observation of the tail fin: moving velocity, heaving cycle and amplitude.

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