Thomann, H. (1959) Aeron. Res. Inst. Sweden (FFA), Rep. 84.
Turner, J.S. (1964) J. Fluid Mech. 18, 195-208.
Vallerani, E. (1964) AE Thesis, CIT.
Van Atta, C. (1966) J. Fluid Mech. 25, 495-512.
Van Atta, C.W. and Helland, K.N. (1980) J. Fluid Mech. 100, 243-255.
van Maanen, H.R.E. (1980) AGARD CP 271, Turbulent Boundary Layers-Experiments, Theory and Modelling, Delft, Paper 3.
Viets, H. and Sforza, P.M. (1972) Phys. Fluids 15, 230-240.
Wygnanski, I. (1970) Israel J. Tech. 9, 105-114.
Wygnanski, I. and Champagne, F. (1973) J. Fluid Mech. 59, 281-335.
Wygnanski, I., Sokolov, M., and Friedman, D. (1975) J. Fluid Mech. 69, 283-304.
Wygnanski, I., Sokolov, M., and Friedman, D. (1976) J. Fluid Mech. 78, 785-819.
IIIII 11111 11111
Wygnanski, I. (1979) Coherent Structure of Turbulent Boundary Layers, Lehigh, 168-193.
Wygnanski, I., Haritonidis, , and Kaplan, R.E. (1979) J. Fluid Mech. 92, 505-528.
Yellin, E.L. (1966) Circ. Research 19, 791-804.
Zdravkovich, M.M. (1969) J. Fluid Mech. 37, 491-496.
Zilberman, M., Wygnanski, I., and Kaplan, R.E. (1977) Phys. Fluids 20, Suppl., S258-S271.

THREE-DIMENSIONAL ASPECTS OF BOUNDARY-LAYER TRANSITION

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1.0 INTRODUCTION

In the present paper it is intended to discuss only a limited number of subjects concerning the three-dimentional aspects of boundary-layer transition, especially those which are not only essential for affording a correct understanding of the basic feature of transition, but also adequate for systematizing the writer's own thoughts on the problem of transition.

2.0 TRANSITION IN TWO-DIMENTIONAL BOUNDARY LAYERS.

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Transition in Blasius boundary layer and plane Poiseuille flow is preceded by the appearance of two-dimentional waves of the type predicted by linear stability theory (Tollmien-Schlichting Waves), but the effect of finite amplitude presents itself in the form of a nearly periodic variation of wave amplitude in the spanwise direction, with maxima (peaks) and minima (valleys) along certain streets parallel to the stream. This variation of wave amplitude generates a system of streamwise vortices, which in turn redistributes the momentum of the basic flow, in particular, producing an inflexional velocity profile at the peak position. The theory of Benney and Lin (1960, 1961, 1964), centered on the non-linear interaction between a two-dimensional wave and a three-dimensional wave with spanwise periodicity, accounts for the generation of streamwise vortices from an initially

weak spanwise variation in wave amplitude, but offers no explanation as to the mechanism by which the three-dimensional effects predominate in the nonlinear wave evolution. The weakly nonlinear stability theory, due primarily to Stuart (1960) and Watson (1960, 1962) for two-dimensional waves, has recently been worked out by Itoh (1980a) to include three-dimensional effects. The theory appears to be successful in indicating the earlier generation of three-dimensional disturbances as the result of a subcritical threshold instability, thus accounting for the experimental evidence (Klebanoff and Tidstrom 1959) for the existence of a threshold amplitude above which the formation of peaks and valleys eventually leads to breakdown of wave motion. On the other hand, the experimental result that increasing the initial wave amplitude beyond the threshold has no significant effect on the
IIIII structure of peaks and valleys appears to
11111 afford a support to the writer's interpre-
22222 tation of the peak and valley formation as
displaying a stable finite-amplitude equilibrium.

Breakdown of wave motion occurs in the form of high-frequency generation of hairpin-shaped eddies, which is interpreted as the onset of a secondary instability of the boundary layer perturbed by the primary instability, exhibiting an inflexional velocity profile for a certain fraction of each cycle of the primary wave at the peak position. The idea of secondary instability of the inflexionally profiled flow, which had long since been anticipated by Prandtl (1933), was worked out first by a crude, quasi-steady, inviscid analysis of Greenspan and Benney (1963), and then by a more ambitious approach of Landahl (1972) to extend the ideas of the kinematic wave theory of Whitham (1974) for conservative systems to slightly dissipative system.

However, Landahl's analysis is difficult to follow without meeting with some inconsistencies. The writer looks forward to a recent attempt of Itoh (1980b) to pursue the way out of the difficulties encountered by Landahl by treating both wavenumber and frequency as complex and introducing the complex space coordinates, with the real coordinate corresponding to the Galilean coordinate moving with the phase velocity of the primary wave. Mention is also made of the boundary layer on a concave wall, where the streamwise vortices (Görtler vortices) are generated as the direct result of primary instability, while the breakdown into hairpin eddies is interpreted as due to the secondary instability of inflexionally profiled boundary layer flow. Importance of Reynolds number is to be noticed for correctly understanding the results of experimental observation such as due to Bippes and
Görtler (1972) and Ito (1980). IIIII

3.0 TRANSITION IN THREE-DIMENSIONAL BOUNDARY 11111
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In a three-dimensional boundary layer such as occurring on a rotating disk or on a swept wing, the velocity vectors along the soild wall possess components both parallel and normal to the inviscid streamline outside the boundary layer. Because of the normal or cross-flow component, there exists a certain range of directions of propogation, in which the velocity profile is unstable and the disturbance is amplified, as first disclosed by Gregory, Stuart and Walker (1955). In the light of the numerical caluclation of Yamashita and Takematsu (1974), the vortex patterns observed prior to transition on a rotating disk are to be intrepreted, however, not as due to the neutral stationary disturbances generated by a particular velocity profile, which changes sign from negetive to positive as the wall is approached and has a point of inflexion at the point of zero velocity as previously surmised, but as due to the most amplified mode of stationary disturbances generated by one of the neighbouring velocity profiles which has a point of inflexion at the point of slightly positive velocity. On the experimental side, Kitamura's observation (1973) deserves special mention from the fact that the disturbance is amplified linearly in the region of $r(\omega/\nu)^{\frac{1}{2}}$ from 4.3×10^2 to 4.9×10^2 and that the nonlinear development of disturbances is characterized by the appearance of the second harmonics and then the third and higher harmonics until eventually random fluctuations prevail.

4.0 TRANSITION INDUCED BY ROUGHNESS ELEMENT

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It is now known from the experiments of Klebanoff and Tidstrom (1972) that the mechanism by which a two-dimensional roughness induces earlier transition is attributed to the destabilizing influence of the flow in the recovery region immediately downstream of the roughness element. There are scattered evidence to imagine that the effect of a three-dimensional roughness element is also stability dominated. Visual observations of Gregory and Walker (1951) and Mochizuki (1961) have revealed the existence of two sets of streamwise vortices downstream of a three-dimensional roughness element at a subcritical free-stream velocity. The one is a closely spaced pair of spiral filaments rising from the wall close behind the roughness until trailing downstream at a level of the top of the roughness, and the other is a horseshoe-shaped vortex filament wrapped round the front of the roughness and trailing downstream. These vortices produce unstable velocity profiles along the center line in the near wake of the roughness, but stable velocity

profiles in the far wake. This appears to explain the experimental observation of the closely spaced vortex filaments breaking up into hairpin eddies with a slight increase in velocity, which, unlike those observed in the transition of Blasius flow on a smooth flat plat, do not directly lead to turbulence. Turbulence might originate somewhere away from the wake center line, where the boundary layer becomes three-dimensional, with the cross-flow velocity component induced by the streamwise vortices. These conjectures are supported by the recent measurements of Gupta (1980) in the wake of a roughness element. It is also probable that the stationary vortices generated by cross-flow instability induce another pair of streamwise vortices downstream and outside until a wedge-shaped turbulent region is formed.

5.0 GROWTH OF A TURBULENT SPOT

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Formation of a turbulent wedge downstream of a roughness element presents a typical example of the spanwise growth of a turbulent region embedded in a turbulent spot, in which the growth normal to the wall is explained by turbulent entrainment. Examination of the profiles of the ensemble-averaged streamwise velocity in a turbulent spot (Handa 1976) suggests the spanwise growth normal to the wall. The spanwise growth appears to be associated with the field of flow which resembles that due to a pair of streamwise vortices in the vicinity of the leading edge of the spot, suggesting that the situation might be similar to that already observed downstream of a three-dimensional roughness element. This corroborates the view that the spanwise growth is due to destabilization by induction of the surrounding fluid, a view shared also by Gad-el-Hak, Blackwelder and Riley in their recent paper (1980).

6.0 MAIN REFERENCES

Benney, D.J. 1961. J. Fluid Mech. 10:209-236

Benney, D.J. 1964. Phys. Fluids 7:319-326.

Benney, D.J., Lin, C.C. 1960. Phys. Fluids 3:656-657.

Bippes, H., Görtler, H. 1972. Acta Mech. 14:251-267.

Cantwell, B., Coles, D., Dimotakis, P. 1978. J. Fluid Mech. 87:641-672.

Charters, A.C. 1943. NACA Tech. Note No. 891

Gad-el-Hak, M., Blackwelder, R.F., Riley,J.J. 1980. Laminar-Turbulent Transition, 297-310, Springer Verlag.

Greenspan, H.P., Benney, D.J. 1963. J. Fluid Mech. 15:133-153.

Gregory, N., Stuart, J.T., Walker, W.S. 1955. Phil. Trans. Roy. Soc. London. Ser. A 248:155-199.

Gregory, N., Walker, W.S. 1951. ARC R. & M. No. 2779, Pt. 1.

Gupta, A.K. 1980. Phys. Fluids 23:221-223.

Handa, N. 1976. Doctor Thesis, Dept. Mech. Eng., Nihon Univ., Tokyo.

Ito, A. 1980. J. Jpn. Soc. Aeron. Space Sci. 28:327-333.

Itoh, N. 1980a. Trans. Jpn. Soc. Aeron. Space Sci. 23:91-103.

Itoh, N. 1980b. To appear in Proc. Roy. Soc. London, Ser. A.

Kitamura, O. 1973. Master Thesis, Dept.Mech. Eng., Hokkaido Univ., Sapporo.

Klebanoff.,P.S. 1966. Proc. 11th Int. Congr. App. Mech., Munich 1964, 803-805, Springer Verlag.

Klebanoff, P.S., Schubauer, G.B., Tidstrom, K.D. 1955. J. Aeron Sci. 22:803-804.

Klebanoff, P.S., Tidstrom, K.D. 1959. NASA Tech. Note D-195.

Klebanoff, P.S., Tidstrom, K.D. 1972. Phys. Fluids 15:1173-1188.

Klebanoff, P.S., Tidstrom, K.D., Sargent, L.M. 1962. J. Fluid Mech, 12:1-34.

Landahl, M.T. 1972. J. Fluid Mech. 56:775-802.

Mochizuki, M. 1961. J. Phys. Soc. Jpn. 16:995-1008.

Orszag, S.A., Kells, L.C. 1980. J. Fluid Mech. 96:159-205.

Prandtl, L. 1933. Z. Ver. Dtsch. Ing. 77:105-114.

Stuart, J.T. 1960. J. Fluid Mech. 9:353-370.

Stuart, J.T. 1962. Adv. Aeron. Sci. 3:121-142, Pergamon Press.

Tani, I. 1973. Z. Angew. Math. Mech. 53:T25-32.

Tani, I. 1980. Laminar-Turbulent Transition, 263-276, Springer Verlag.

Tani, I., Komoda, H., Komatsu, Y., Iuchi, M. 1962. Aeron. Res. Inst., Tokyo Univ., Rep. No. 375.

Tani, I., Sato, H. 1956. J. Phys. Soc. Jpn. 11:1284-1291.

Watson, J. 1960. J. Fluid Mech. 9:371-389.

Watson, J. 1962. J. Fluid Mech. 14:211-221.

Whitham, G.B. 1974. Linear and Nonlinear Waves, John Wiley & Sons.

Yamashita, I., Takematsu, M. 1974. Res. Inst. App. Mech., Kyushu Univ., Ren. No. 69.