A MODEL OF THE TURBULENT BURST PHENOMENON: PREDICTIONS FOR THE NUMERICAL P-FUNCTION

M.A. El-Hadidy, M.B. Ibrahim, and L.C. Thomas
University of Petroleum and Minerals
Dhahran, Saudi Arabia

1.0 INTRODUCTION

Numerical computations of momentum and heat transfer is turbulent wall flows have been given a great deal of attention in the last two decades. These flows are characterized by a large cross-stream variation in the local mean velocity $\bar{u}$, temperature $\bar{T}$, and turbulent Reynolds number $\rho K^{\frac{1}{2}}\ell/\mu$, where K, ℓ, , and μ represent turbulent kinetic energy, turbulence length scale, density, and laminar viscosity, respectively. The turbulent Reynolds number increases sharply from zero at the wall to a large value within a very thin layer adjacent to the wall. The thickness of this wall region depends on the mean flow velocity, the fluid properties, and the geometry. It is the wall region which represents one of the major problems in momentum and heat transfer computations. This is due to the difficulties encountered in modeling the turbulent characteristics (Reynolds stress $\bar{\tau}_t$, turbulent Prandtl number Pr_t, etc.) which are affected in the region by unsteady molecular transport.

Whereas some progress has recently been achieved in the development of one- and two-equation turbulence models for fluid flow in the region close to the wall/1-6/, standard numerical computation schemes currently available generally avoid numerical computations in this important region by utilizing simplifying approximations (i.e., F

wall-functions)/7/. For heat transfer the semi-empirical P-function, which accounts for the thermal resistance of the laminar sublayer, has been employed with the turbulent Prandtl number Pr_t taken as a constant; the P-function is defined in terms of distributions in the dimensionless mean velocity $u^+(\equiv \bar{u}/U^*)$ and temperature $T^+[\equiv (T_o - \bar{T})\rho c_p U^*/\overline{q''_o}]$ by the equation

$$T^+ = Pr_t(P + u^+) \quad (1)$$

An alternative approach to analyzing the important wall region has been developed over the past few years which is based on the surface renewal model of the turbulent burst phenomenon/8-11/. In this approach, the unsteady molecular transport of momentum and energy associated with the turbulent burst process is modeled. In order to incorporate this approach to modeling wall turbulence into the framework of modern numerical methods for computing turbulent convection heat transfer, a generalized form of the surface renewal model (known as the surface rejuvenation model) is utilized in the present paper to develop a theoretical relationship for the P-function.

2.0 THE MODEL OF WALL TURBULENCE

A comprehensive physical picture of the burst process involves packages of fluid which move from the turbulent core to within various close distances H of the wall. After residing within the wall region for brief lengths of time, these fluid elements are ejected back into the turbulent core. This process is illustrated in Fig. 1.

The differential formulation for the instantaneous transfer of energy within an individual element of fluid near the wall between the inrush and ejection phases of a

burst event are given by (assuming uniform property conditions and negligible convective and pressure gradient effects)/12/

$$\frac{\partial T}{\partial \theta} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (2)$$

and intitial-boundary conditions $T(o,y)=T_i+[h(y)-T_i]U(y-H)$, $T(\theta,o)=T_o$, and $T(\theta,\infty)=T_i$; θ is the instantaneous time, T is the local instantaneous temperature distribution, T_o is the local instantaneous wall temperature, T_i is the temperature of the inrushing fluid, h(y) is the instantaneous temperature distribution within the wall region at the instant of inrush, H is the instantaneous approach distance, and U(y-H) is a unit step function (see Fig. 1). Following the procedure outlined in Reference 12, these equations are transformed into the mean domain by utilizing a statistical distribution in θ, with the result

$$(\bar{T}-T_i)\left[1 - \exp(-\frac{y}{\bar{H}})\right] = \frac{\alpha}{\bar{s}} \frac{d^2\bar{T}}{dy^2} \quad (3)$$

and $\bar{T}(o)=\bar{T}_o$ and $\bar{T}(\infty)=T_i$, where $\bar{T}_o$ is the local mean wall temperature, and $\bar{s}$ is the mean frequency of the turbulent burst process.

This system of equations has been solved for $\bar{T}$ to obtain/12, 13/

$$T^+ = 2H^+ \frac{Pr}{\beta} \frac{J_{2\beta}(2\beta) - J_{2\beta}\{2\beta \exp[-y^+/(2H^+)]\}}{J_{2\beta-1}(2\beta) - J_{2\beta+1}(2\beta)} \quad (4a)$$

where $\beta=\gamma\sqrt{Pr}$ and $\gamma=\bar{H}\sqrt{\bar{s}/\nu}$. Based on the fluid flow analysis presented in reference 12, γ has been set equal to 0.433, such that $\bar{s}=7.52\times10^{-3}U^{*2}/\nu$. An approximate second order interface has been established between Eq. (4a) and the logarithmic inner law

$$T^+ = \frac{1}{\kappa} \ln y^+ + A \qquad \text{for } y^+ > y_H^+ \tag{4b}$$

where $\kappa \simeq 0.4$. The resulting predictions for β and the interface location y_H^+ are given in Table 1.

The dimensionless mean temperature distribution T^+ reduces to u^+ for Pr equal to unity; that is

$$u^+ = \frac{2H^+}{\gamma} \frac{J_{2\gamma}(2\gamma) - J_{2\gamma}\{2\gamma \exp\left[-y^+/(2H^+)\right]\}}{J_{2\gamma-1}(2\gamma) - J_{2\gamma+1}(2\gamma)} \qquad y^+ < 35 \tag{5a}$$

$$= \frac{1}{\kappa} \ln y^+ + 5.5 \qquad y^+ > 35 \tag{5b}$$

Utilizing Eqs. (4b) and (5b) together with the first order approximation $Pr_t \simeq 1$, the P-function in the region outside the wall region is given by

$$P = A - 5.5. \tag{6}$$

The wall turbulence model predictions for A are coupled with this equation to produce the theoretical P-function curve shown in Fig. 2. The figure also includes the previously recommended empirical correlations by Launder and Spalding/7/and Jayatilleke/14/which have been widely used in the numerical computation of heat transfer characteristics. The theoretical predictions for the P-function are seen to be in fairly good agreement with these empirical correlations.

3.0 CONCLUSION

A new P-function has been developed for adaptation to the numerical computation of heat transfer characteristics in turbulent wall flows. The development is based on the statistical surface rejuvenation model of wall turbulence. The present model predictions for the P-function are in good agreement with the previously developed correlations. Because of its strong theoretical base, the present method is believed to provide a basis for handling more complex heat transfer problems (such as variable property and transitional turbulent flows.) The P-function developed in this paper is presently being used with the two-equation (K-ε) model of turbulence in the computation of heat transfer in turbulent pipe flow. This work will be reported on soon.

4.0 REFERENCES

/1/ Wolfshtein, M. (1969), Int. J. Heat and Mass Transfer, 12, 301-318.

/2/ Jones, W.P., and Launder, B.E. (1972), Int. J. Heat and Mass Transfer, 15, 301-314.

/3/ Jones, W.P., and Launder, B.E. (1973), Int. J. Heat and Mass Transfer, 16, 1119-1130.

/4/ Hassid, S., and Poreh, M. (1975), J. Fluid Eng., 97, 234-241.

/5/ El Hadidy, M.A. and Spalding, D.B. (1979), Imperial College, Mech. Eng. Dept., Heat Transfer Section Report No. HTS/79/9.

/6/ El Hadidy, M.A. (1979), Ph.D Thesis, Imperial College, Mech. Eng. Dept., Heat Transfer Section.

/7/ Launder, B.E., and Spalding, D.B.

(1973), Imperial College, Mech. Eng. Dept., Heat Transfer Section Report No. HTS/73/2.

/8/ Hanratty, T.J. (1956), AIChE J., 2, 359.

/9/ Thomas, L.C. (1970), J. Heat Transfer, 92, 565.

/10/ Katsibas, P., and Gordon, R.J. (1974), AIChE J., 20, 191.

/11/ Rajagopal, R., and Thomas, L.C. (1974), Fifth Int. Heat Transfer Conf., Tokyo.

/12/ Thomas, L.C. (1980), Int. J. Heat Mass Transfer, in press.

/13/ Thomas, L.C., Gingo, P.J., and Chung, B.T.F. (1975), Chem. Eng. Sci., 30, 1239.

/14/ Jayatilleke, C.L.V. (1969), Progress in Heat and Mass Transfer, 1, 193.

TABLE I

Pr	A	y_H^+
0.72	2.6	36
1	5.5	35
2	13.8	30
5	31.9	24
10	54.3	20
25	104	15
100	266	10
200	411	8.1
300	551	7.46

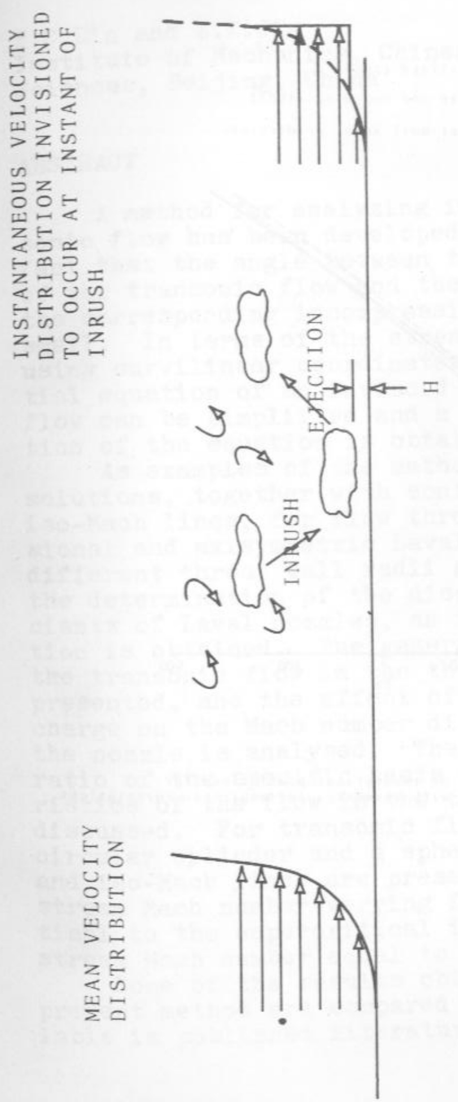

Figure 1. Picture of turbulent burst process.

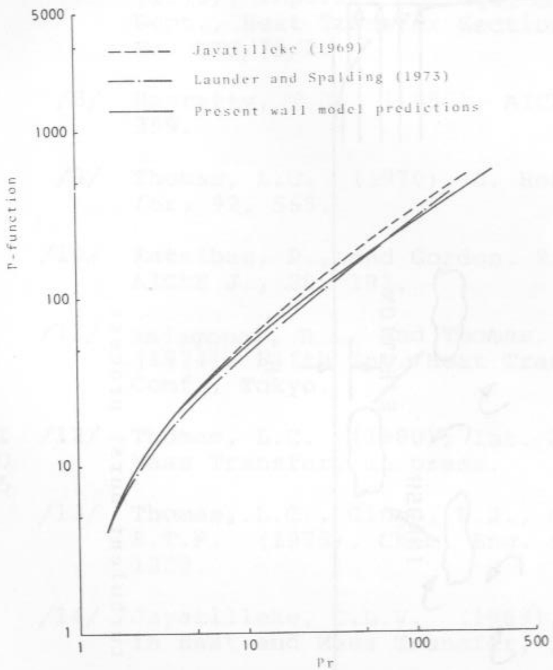

Figure 2. Comparison of theoretical predictions for P-function with commonly used empirical correlations.

IIIII
00000
55555

ANALYSIS OF TRANSONIC INVISCID FLOW

T.C.Lin and Z.X.Jia
Institute of Mechanics, Chinese Academy of Sciences, Beijing, China

ABSTRACT

A method for analyzing inviscid transonic flow has been developed, based on the fact that the angle between the streamline of the transonic flow and the streamline of the corresponding incompressible flow is small. In terms of the stream function, on using curvilinear coordinates, the differential equation of an inviscid compressible flow can be simplified and a general solution of the equation is obtained.

As examples of the method, transonic solutions, together with sonic lines and iso-Mach lines, for flow throuth two-dimensional and axisymmetric Laval nozzles of different throat wall radii are given. For the determination of the discharge coefficients of Laval nozzles, an integral relation is obtained. The general behaviour of the transonic flow in the throat region is presented, and the effect of the mass discharge on the Mach number distribution in the nozzle is analysed. The effect of the ratio of the specific heats on the characteristics of the flow in the throat region are discussed. For transonic flow around a circular cylinder and a sphere, sonic lines and iso-Mach lines are presented for free stream Mach number varying from the subcritical to the supercritical including free stream Mach number equal to one.

Some of the results obtained by the present method are compared with those available in published literature. It shows

IIIII
00000
66666