GALAXIES, TURBULENCE, AND PLASMAS

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The three exciting areas of research named in the title, apparently disjointed in contents, have similar basic mechanisms in common, which can be described by the same mathematical principles, concepts and methods. In this article, we shall discuss or refer to scientific problems from all three areas, with emphasis placed on galaxies, where observational data are plentiful for checking the theory. We shall now begin our discussions with the theory of the spiral structure of galaxies, and we shall see that the subject is closely related to the study of hydrodynamic stability and turbulence.

A galaxy is essentially a collection of stars. The system of basic equations for the description of the dynamics of stars is essentially of the same nature as those for the description of electromagnetic plasmas.

Galaxies exhibit a variety of morphological appearances: elliptical, spiral, barred spiral, and irregular. A galaxy is usually thought of as observed in optical frequencies, but it may also be observed through radiations at other frequencies, e.g., radio, X-ray, and gamma-ray. The spiral structures observed at optical and at radio frequencies are usually quite similar.

An important class of galaxies has a disk-like component, with a nucleus in the middle. Such a system must be rotating. Otherwise, self-gravitation would have pulled it together. It turns out that the inner part is rotating faster than the outer part, in such a manner that the linear velocity of rotation is nearly constant. This is known as shearing motion in hydrodynamics, and as differential rotation in astrophysics. In such a field of motion, any material clump would be stretched out into a part of a trailing spiral structure. "But this is not the phenomenon we must consider. We must consider a spiral structure over the whole galaxy from the nucleus to its outermost part, and consisting of two arms starting from diametrically opposite points. Although this structure is often hopelessly irregular and broken up, the general form of the large scale phenomenon can be recognized in many nebulae [galaxies]." (Quotation from paper by Jan Oort, 1962) This issue is often referred to as that of the existence of grand design.

Another important issue is that of the winding dilemma. To describe this issue, we have to state some observational facts. Non-barred spiral galaxies (or normal spiral galaxies) are classified by Hubble according to the tightness of winding into Sa, Sb, and Sc spirals, Sc being the most open. You might imagine that because of differential rotation, Sc galaxies would soon wind toward Sa, because the inner part is rotating faster, and like a spool of string, would therefore tend to become tighter and tighter with rotation. But this is not observed to be the case. Of course we cannot directly follow the evolution of galaxies in our lifetime: this winding would occur on the order of a few hundred million years. However, we can make a statistical study and show that Sc galaxies and Sa galaxies are physically different through the observation of other physical characteristics; for example, the gas content in Sc is much higher than in Sa. You can say that Sc galaxies would have their gas formed into stars and then become Sa at the same time. But if that were so, the average mass and the number of stars formed would be so large that Sc galaxies would be much more brilliant than they actually are. Furthermore, the mass distribution is such that there is a very small nucleus in Sc galaxies whereas Sa galaxies have more massive nuclei. It is impossible for mass to accumulate so rapidly because the angular momentum in the system cannot be adjusted so quickly. Thus, the evolution from Sc to Sa in a reasonable period of time is ruled out, and they must be rather permanent structures. The question is: If we have material objects arranged like that in an Sc galaxy, why does it not wind down to an Sa structure? This is the so-called winding dilemma.

The answer is, as it turns out, that Sc and Sa galaxies have their large scale spiral structure in the form of permanent or nearly permanent wave patterns. These patterns have now been calculated by using a number of

methods and the mechanisms for their maintenance have been understood. Indeed, it appears to be promising to use the spiral patterns calculated as a reasonable basis of a dynamical approach to the morphological classification of galaxies.

Waves over a system in differential rotation are well-known in the study of turbulence. Theory of instabilities of this kind goes back to Lord Rayleigh, in 1880, and has been developed over the years. There were mathematical difficulties, so the theory was not fully developed until much later. There were also experimental difficulties, so the theoretical predictions were not experimentally checked until the 1940's. More recently in Japan, experimentalists have also checked the calculations for the classical case of flow through a two-dimensional channel. Thus, we are applying these well-known concepts of waves of permanent form over a flow system in shear (in differential rotation) to the study of galaxies. The study of wave trains and wave packets in shear flow is also of basic importance in other sciences such as meteorology and oceanography.

Observations show that the spiral structures in galaxies are especially associated with brilliant bluish stars. How can they, as material objects, always stay with the wave? The answer turns out to be quite simple. They are, as a matter of fact, like the white caps on the ocean: they come and go, they are formed and then they disappear. They are now believed to form out of the interstellar medium (the gas) and then shine brilliantly by burning their nuclear fuel. After exhausting their nuclear fuel, they disappear with a bang, a supernova explosion. Can these things happen over the time period under consideration? Indeed the answer is: Yes! For the time scale for the evolution of such brilliant stars into the supernova state and then into the white dwarfs is one to ten million years, and the time of one period of revolution of the galaxy is about 200 million years. So it is during a small fraction of a period of revolution of the galaxy that the whole phenomena of star formation and star disappearance can occur, and they are no more permanent than the white caps at the crest of waves on the ocean. This is another example where the concepts used to explain the phenomena of turbulence, hydrodynamics, and galaxies get together. The scenario described above has been checked through detailed comparison of theoretical predictions and observations.

We now turn to the discussion of some similarities in mechanisms among the three subjects under discussion: galaxies, turbulence, and plasmas. In this extended abstract, we shall just list some of the relevant items.

1. As mentioned at the beginning, the basic equations in plasma physics (the Vlasov equations) are of the same nature as those for the study of stellar systems.

2. Both in the study of hydrodynamic stability and in the study of spiral waves in galaxies, the critical layer (or the corotation circle in the case of galaxies) plays an important role in the transfer of energy between the waves and the basic motion. Indeed, the behavior of wave trains and wave packets in this region, involving the phenomenon of over-reflection (or WASER) is of basic importance to a number of natural phenomena.

3. The WASER mechanism (wave amplification by stimulation of emission of radiation) is important for spiral wave patterns in galaxies, as well as in plasma dynamics and in the instability of supersonic shear layers. It includes the interaction of waves of positive and negative energy densities.

4. There is similarity (but there is also essential difference) between the density waves in galaxies and the Bernstein waves in magnetically contained plasmas. This is due to the similarity between the Coriolis force in the former case and the Lorentz force in the latter case. The law of inverse square holds in both cases.

5. Analogous mechanisms may also be found between the instability of ballooning modes in magnetically contained plasmas and the instability of Couette flow between rotating cylinders, with the inner cylinder rotating.

The fact that diverse scientific problems exhibit similar behavior is indeed the basis for the possibility to have them treated by similar mathematical concepts and methods. Indeed, by providing a unified mathematical approach to diverse scientific problems, one can hope to gain deeper understanding — e.g., on the matter of interaction of waves of positive and negative energy. This is indeed one of the principal roles of the applied mathematician.

SEDIMENTATION : A REVIEW OF DEVELOPMENTS IN A CLASSICAL PROBLEM

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The term 'sedimentation' conventionally refers to a large number of small particles falling under gravity through fluid which is otherwise at rest. Sedimentation is used often in chemical and biological laboratories, perhaps with an ultracentrifuge giving an enhanced gravitational force, as a means of separating the particles and the fluid. It is also used widely in chemical industry and water engineering and mining for the same purpose of separation, and in chemical industry the related process of fluidization is exploited to achieve a high rate of mass or heat transfer between the particles and the fluid. In nature we see sedimentation of ash, dust and water droplets in the atmosphere, silt particles in the rivers and oceans, and fat globules in milk. A full understanding of the dynamical problems involved in sedimentation of small particles would consequently have considerable practical value. The total research effort over the years has been massive, but it has proved to be difficult to obtain quantitative theoretical results, mainly because we lack suitable methods of analysing the motion of many interacting particles. However, a variety of intriguing phenomena have been revealed and a qualitative understanding of most (but not all) of the relevant processes has been obtained.

The purpose of this lecture is to take stock of the developments in basic aspects of sedimentation of particles smaller than about 0.1mm in diameter, and to review, in fundamental terms, our present picture of the dynamical processes involved. Many of the developments are recent, and for those interested in