# General Lecture

## On the Theory of Turbulence for Incompressible Fluids

By

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Abstract

The theory of turbulence for incompressible fluids based upon Reynolds' equations of mean motion and the dynamical equations of turbulent velocity correlations was first developed by the author in the two papers published in the Chinese Journal of Physics in 1940 and in the Quarterly of Applied Mathematics in 1945. The equations of correlations are built from the equations of turbulent velocity fluctuation which are the differences of the Navier-Stokes equations and the Reynolds equations. Due to the non-linearity of the equations of turbulent fluctuation, the equations of velocity correlations thus built are not closed. The author put forward in his 1940 paper the relation between the velocity correlations of the fourth and second orders together with other hypotheses to render the systems of the dynamical equations closed. Since then papers along this line of approach have been published by Rotta (1951), Davidov (1961), Donaldson (1969), Daly and Harlow (1970), Kolovandin and Vatutin (1972), Hanjalić and Launder (1972), and others. The main point of these papers is to assume different relations between the velocity correlation of a given higher order with that of a lower order like what the author did in 1940. This is known as the "closure program".

But this "program" is not satisfactory for two reasons: In the first place, such a relation must have experimental basis; secondly, although we can obtaim closed systems of correlation equations thus built and their solutions to compare with observations, but experiments can be performed to measure velocity correlations of still higher orders which are beyond this "program".

In the XVth International Congress of Theoretical and applied Mechanics held in Toronto and in the First Asian Congress of Fluid Mechanics held in Bangalore both in 1980, the author brought up the "condition of pseudo-similarity" in the paper on the theory of homogeneous isotropic turbulence, leading to results in agreement with a large number of experiments. This condition is now generalized to the general shear flows to supplement the system of dynamical equations derived before. The method of solving the turbulence problem by seeking the solutions of the Reynolds equations and the correlation equations of successive orders is now considered as a mathematical method of successive approximation. We can truncate this process and stop at the order of the correlation we need for solving a given turbulence problem.

This process is used, as the first stage of approximation, to solve the turbulent flows like the pressure flow within a channel, the plane turbulent wake, and the plane and axial jets, as examples. In this stage equations of the mean motion and of the double velocity correlations are used, while the third order velocity correlation and its equations are ignored. Agreements between experiments of the mean velocities and double velocity correlations with the corresponding calculated values are satisfactory. We note that Kármán's law of similarity for the channel flow follows as a first approximation in this stage.

The second stage of this process consists of deriving the dynamical equations of the velocity correlations of the triple and quadruple orders, and seeking their solutions together with the equations of mean motion and of the double velocity correlation, while correlations of the fifth order and their equations are neglected. This process is applied to the solution of the plane wake and calculations are underway.