THE FOURTH ASIAN CONGRESS OF FLUID MECHANICS

August 21-25, 1989 Hong Kong

TRANSIENT SHAPE OF THE FREE SURFACE OF A LIQUID IN AN ABRUPTLY-ROTATING CYLINDER

Sangmin CHOI [+], Jae Won KIM and Jae Min HYUN

[+] School of Mechanical Engineering, KIT, Korea
Department of Mechanical Engineering, KAIST, Korea

ABSTRACT An experimental investigation was made of the impulsive spin-up from rest of a liquid in a partially-filled cylindrical container. The main impetus was placed on delineating the effects of the presence of a free surface which intersected either or both of the endwall disks during the course of spin-up. Extensive flow visualizations were carried out, and the propagating velocity shear front and the shape of free surface were identified. An image processing technique was utilized to determine accurately the propagating velocity shear front as well as the time-dependent free surface contour. This technique was verified by the precise velocity measurements using a laser Doppler velocimeter. The experimentally determined free surface contour and the radial location of the propagating shear front were found to be consistent with the previous analytical predictions.

1. Introduction

One problem of central importance in modern rotating fluid dynamics is the issue of spin-up from rest[1,2,3]. Consider a flow configuration in a vertically-mounted circular cylinder (radius R, height H, see Fig. 1 for the flow system) at rest. The fluid in the circular cylinder is viscous, incompressible and the cylinder is partially filled. At a certain instant t=0, the container is impulsively started spinning about its longitudinal axis, aligned parallel to gravity, at a finite constant angular speed Ω. Here, the main interest is confined to the classes of flow for which the rotational Reynolds number, $Re = \Omega R^2/\nu$, is sufficiently large. The transient fluid motions will persist until the entire body of the liquid acquires the same angular speed as that of the solid boundaries of the container. This flow situation is highly relevant to such practical engineering applications as chemical mixers, centrifuges, and fluid machinery, etc.

The bulk of the previous studies on spin-up was concerned with the situations in which the cylindrical container was filled completely with a liquid[1,4,5,6]. These previous works have measurably deepened our understanding of the principal dynamic ingredients involved in the process. The essential result of the works is that, since $Re \gg 1$, the entire flow field may be divided into two regions, i.e., the essentially inviscid interior region and the boundary layers enclosing the inviscid interior core. In the interior, the horizontal velocity(u) and the pressure(p) are substantially independent of the axial position(z), resulting in the "columnar" flow approximation. The Ekman boundary layers, which form on the endwall disks perpendicular to the rotation axis, heavily influence the flow in the interior region. The meridional circulation to transport angular momentum from the rotating boundaries to the interior is caused by the Ekman layer pumping due in the layer. It shortens the transient time of spin-up process in comparison with the flow situation in which the only mechanism of the angular momentum transfer is the viscous diffusion. Recent investigations employed advanced experimental and numerical methodologies to reconfirm the previous analytical results.

Recently, the question of spin-up of a liquid in a partially-filled cylindrical cavity has emerged to be of considerable concern. This flow configuration shows the angular momentum transfer process affected by the free surface. It is natural to raise a question as to what extent the free surface effects the spin-up process. A previous work of Homicz and Gerber[7] classified the free surfaces as four cases according to the shape of the free at steady state. There were two primary parameters to determine the final-state of the free surface; one is the Froud number, $Fr=(\Omega R)^2/gH$ and the other the fill-ratio that represents the mount of the liquid in the container at rest. Also, they developed a simplified numerical code based on the work of Wedemeyer[1]. The analytical model of Ref. [2] had physically plausible assumptions and was capable of depicting the transient flow field when the free surface intersects either or both of the endwalls. The results of Ref. [2] were experimentally verified, only when the free surface did not intersect any endwalls, by the work of Goller and Ranov[8]. The primary purpose of this study is to validate experimentally the predictions of the preceding analytical model, with particular reference given to the model of Ref. [2], to deal with the cases when the free surface intersects either or both of the endwall disks.

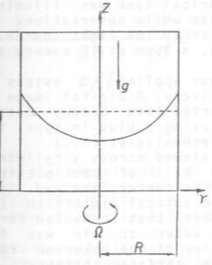

Fig. 1. Schematic diagram of the flow system.

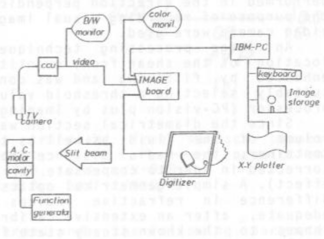

Fig.2. Experimental set-up.

2. The experiment

A carefully controlled experimental program was undertaken to measure the free surface shape and the radial location of the propagating velocity shear front. The experimental apparatus consisted of a rotating turntable, cylindrical containers, a visualization set-up, an image processing system and a flow velocity measuring device. Figure 2 is a schematic of the experimental rig.

A rotating turntable was built with a precision controlled AC servo motor (0.4 kw. rating). The rotational speed of the turntable was adjusted by varying the input pulse signal. The speed was continuously checked by monitoring the speed signal from the servo unit. The table was able to reach the steady rotation within 0.0012 seconds(i.e., less than 0.1 revolution at the typical rotation speed 400 rpm) and variation of the rotating speed was less than ±0.05 percent under the normal operating condition of 400 rpm. A series of selected cylindrical tanks were fabricated with plexiglass. The size of these tanks was chosen so that the desired aspect ratios H/R could be duplicated. The cylindrical container was coupled to the table with special care to guarantee the upright configuration. Typical working fluid was water at 18(±0.5) degree centigrade.

An observation of the contour of the free surface was made by using reflections from a fluorescence-dye enhanced fluid. The diametrical cross section of the free surface was visualized by a planar light source. A slit beam generator, equipped with a 1500 watt halogen lamp, was used to excite the fluorescence-dye dissolved in water. The wavelength of reflection from the fluorescence-dye was in the visible range.

Two complementary techniques were utilized in order to portray the velocity shear front; image processings of the flow visualization data and velocity measurements by way of laser Doppler velocimeter. The radial location of the propagating velocity shear front was visualized by using aluminum powder suspended in the fluid, which was a classical technique as described in Greenspan[9]. The cylindrical tank was illuminated from the side with a vertical sheet beam while observations were performed in the direction perpendicular to the light source. For the purpose of recording visual images, a 35mm still camera and a video camera were used.

An image processing technique was applied to assess the location of the shear front quantitatively. A digital image was enhanced by filtering and was converted into binary data by suitably selecting a threshold value using a plug-in type image processor (PC-vision plus by Imaging Technologies Inc.).

Since the diametrical section was viewed across a cylindrical volume of the fluid as well as the wall of the plexiglass container, the radial distance from the center line had to be corrected in oder to compensate for the optical distortion (lens effect). A simple geometrical optics model that accounted for the difference in refractive indices of water to air was found adequate, after an extensive calibration of the observed radial shapes to the known steady state free surface contours. This model was incorporated in the subsequent quantifications of the

cross-sectional image data.

Circumferential velocity measurements in a rotating cylinder were previously reported[3, 10]. A similar procedure was followed with a TSI Model 1990 laser Doppler velocimeter, and a 1988 timer/counter unit with an interface to an IBM-PC/XT compatible. However, care was exercised in order to determine the location of the focal point (see e.g. 10, 11). The water inside the container was seeded with a small amount of milk. An optimum seeding was found to be 1 to 2000 in volume ratio.

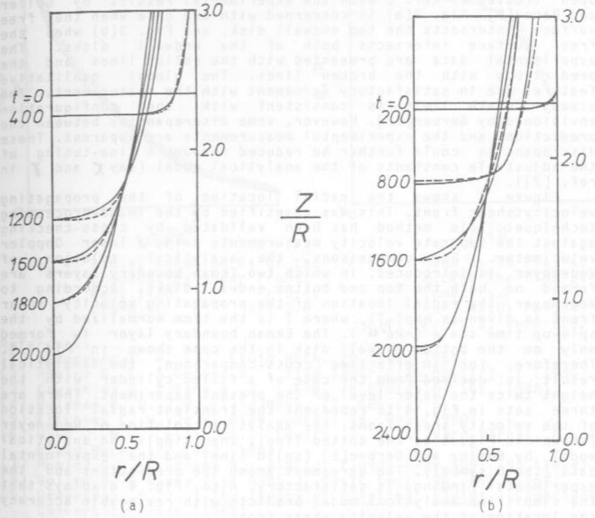

Figure 3. Comparisons of the transient free surface shapes, ———— experiment; - - - - - -, prediction.
(a) $Re=1.2x10^5$, $Fr=3.0$, $H/R=3.0$, and $L/H=0.8$.
(b) $Re=1.2x10^5$, $Fr=5.5$, $H/R=3.0$, and $L/H=0.8$.

3. Results and discussion

The experiments were performed under the following parameter ranges: $Re \sim O(10^5)$, $Fr \sim O(1)$, aspect ratio, $H/R \sim O(1)$. These values were selected so that a direct comparison to the analytical predictions[2] would be possible. In presenting the results, we shall concentrate on the two main points, i.e., the contour of the transient free surface and the radial location of the propagating velocity shear front.

Figure 3 exemplifies the contour of the free surface during the transient process of spin-up in a partially filled cylinder. The flow configuration in which the free surface does not intersect any endwalls is omitted because that case has already been treated in Ref. 2 with the experimental results by Goller and Ranov[8]. Fig. 3(a) is concerned with the case when the free surface intersects the top endwall disk, and Fig. 3(b) when the free surface intersects both of the endwall disks. The experimental data are presented with the solid lines and the predictions with the broken lines. The global qualitative features are in satisfactory agreement with the measurements. The steady state limit is consistent with the configuration envisioned by Gerber[12]. However, some discrepancies between the predictions and the experimental measurements are apparent. These discrepancies could further be reduced through a fine-tuning of the adjustable constants of the analytical model (say κ and γ in ref. [2]).

Figure 4 shows the radial location of the propagating velocity shear front. This was quantified by the image-processing technique. This method has been validated by cross-checking against the separate velocity measurements using a laser Doppler velocimeter. For comparisons, the analytical solution of Wedemeyer is introduced, in which two Ekman boundary layers are formed on both the top and bottom endwall disks. According to Wedemeyer, the radial location of the propagating velocity shear front is given as $\exp(-T)$, where T is the time normalized by the spin-up time scale ($=Re^{1/2}\Omega^{-1}$). The Ekman boundary layer is formed only on the bottom endwall disk in the case shown in Fig. 4. Therefore, for an effective cross-comparison, the analytical result is derived from the case of a filled cylinder with the height twice the water level of the present experiment. There are three sets in Fig. 4 to represent the transient radial location of the velocity shear front; the analytical solution of Wedemeyer (shown in Fig. 4 as the dotted line), the simplified analytical model by Homicz and Gerber[2] (solid line) and the experimental data (the O symbol). The agreement among the predictions and the experimental findings is satisfactory. Also, Fig. 4 displays that the simplified analytical model predicts with reasonable accuracy the location of the velocity shear front.

An extreme case would be when the free surface intersects both the top and bottom endwall disks. Fig. 5 compares satisfactorily the analytical predictions and the experimental measurements ; the symbol(circle) represents the experimental results and the solid line the analytical prediction by Ref. [2]. Some discrepancy appears around the time twice the spin-up time scale($=t/Re^{1/2}\Omega^{-1}$). This means that the deformation of the free surface is nearly completed at that time.

The comparison between the analytical prediction and the experimental measurement shows that the simplified predictions of

interior azimuthal velocity structure are described accurately by the method of Ref. [2]. Also, the results indicate that the spin-up process is, in general, retarded when a free surface is present. The details of these findings will be reported in full-length papers [13,14].

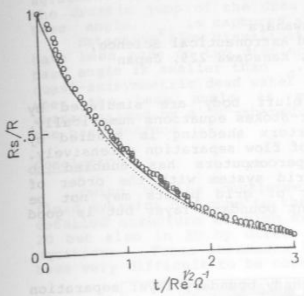

Fig. 4. Radial location Rs of the propagating shear front. $Re=1.1x10^5$, $Fr=0.4$, $L/H=0.5$, $H/R=3.0$.

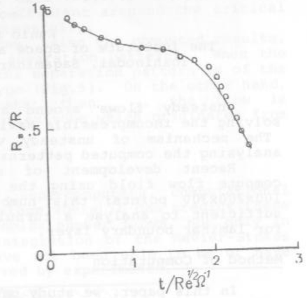

Fig. 5. Same as in Fig. 4, except the flow parameters. $Re=3.4x10^5$, $Fr=3.4$, $L/H=0.5$, $H/R=3.0$.

References

[1] Wedemeyer E. H. : J. Fluid Mech. 20, 383 (1964).
[2] Homicz G. F. & Gerber N. : AIAA paper AIAA-86-1121, 1 (1986).
[3] Kawashima G. & Yang W.-J. : Exp. in Fluids 6, 165 (1988).
[4] Weidman P. D. : J. Fluid Mech. 77, 685 (1975).
[5] Kitchens C. W. Jr. : AIAA Journal 18, 929 (1980).
[6] Hyun J. M., Leslie, F., Fowlis, W. W. & Warn-Varnas, A.: J. Fluid Mech. 127, 263 (1983).
[7] Homicz G.F. & Gerber N.: ASME J. Fluids Engr.109, 195 (1978).
[8] Goller H. & Ranov T. : ASME J. Basic Engr. 90, 445 (1968).
[9] Greenspan H. P. : The Theory of Rotating Fluids, Cambridge University Press (1968).
[10] Watkins W.B. & Hussey R.G. : Phy. of Fluids 20, 1596 (1977).
[11] Boadway J. D. & Karahan E.: DISA Information 26, 10 (1981).
[12] Gerber N. : ASME J. Applied Mech. 97, 734 (1975).
[13] Choi S, Kim J. W. & Hyun J. M.: submitted to ASME J. Fluids Engr. (1988).
[14] Choi S, Kim J. W. & Hyun J. M.: submitted to ASME J. Fluids Engr. (1989).