Proceedings of Eighth Asian Congress of Fluid Mechanics
December 6-10, 1999, Shenzhen, China

CURRENT STATUS OF KINETIC NUMERICAL SCHEMES

S M Deshpande
(CFD Centre, Department of Aerospace Engineering, Indian Institute of Science,
Bangalore – 560 012, India)

ABSTRACT: The Kinetic Numerical Schemes developed in the CFD Centre over a period of ten years have been reviewed. The research ideas described in this paper have been successfully carried over to industries for the computation of flow over many practical configurations of interest to Indian Aerospace Laboratories and Agencies.

I. INTRODUCTION

Kinetic Numerical Schemes use the connection between the Boltzmann equation of Kinetic theory of gases and Euler equations of gas dynamics or Navier-Stokes equations governing the dynamics of compressible viscous fluid. It is well known that suitable moments (called ψ - moments) of the Boltzmann equation give Euler equations when the velocity distribution function is a local Maxwellian (F) and Navier-Stokes equation when the distribution function is Champan-Enskog distribution (f_{CE}). Any numerical scheme for the Boltzmann equation reduces to a mapped numerical scheme for these equations when moments are taken. Such numerical schemes are called kinetic numerical schemes and this strategy is called moment method strategy by Deshpande [1]. This idea has been vigorously pursued in CFD laboratory of Indian Institute of Science, Bangalore and has led to a variety of kinetic schemes known as Kinetic Flux Vector Splitting (KFVS)[2], Peculiar Velocity based Upwinding (PVU)[3], KFVS based on entropy variables (q-KFVS)[4,5], Kinetic Smoothed Particle Hydrodynamics (KSPH)[6], Kinetic Flux Vector Splitting on Moving Grids (KFMG)[7,8] and finally these flux vector splitting combined with Least Squares Kinetic Upwind Method (LSKUM) and therefore leading to a variety of schemes called LSKUM-KFVS and LSKUM-PVU[9,10], LSKUM-MG[11] which are capable of operating on arbitrary meshes or just a distribution of points. This paper is concerned primarily with the current status of kinetic schemes developed in CFD Laboratory.

II. BASIC THEORY

The basic principles are best illustrated by considering 1-D Boltzmann equation

$$\frac{\partial F}{\partial t} + v\frac{\partial F}{\partial x} = 0 \qquad (1)$$

where the collision term has been dropped as it does not matter for our problem. Here f= velocity distribution function, v = particle (or molecular) velocity. The moments of this equation yield the 1-D Euler equation

$$\frac{\partial U}{\partial t} + v\frac{\partial G}{\partial x} = 0 \qquad (2)$$

whenever f = F = local Maxwellian = $\rho\sqrt{\frac{\beta}{\pi}}\exp\left[-\beta\left(v-u^2\right)-\frac{I}{I_o}\right]$ Here ρ=mass density, $\beta=1/(2RT)$, R = Gas Constant, T = Temperature, u = fluid velocity and I_o = internal energy corresponding to non translational degree of freedom. The flux vector splitting is obtained by taking moments of CIR split Boltzmann equation

$$\frac{\partial F}{\partial t} + \frac{v+|v|}{2}\frac{\partial F}{\partial x} + \frac{v-|v|}{2}\frac{\partial F}{\partial x} = 0 \qquad (3)$$

We then get the following expressions for split fluxes,

$$G^{\pm} = \begin{bmatrix} \rho u A^{\pm} \pm \left[\dfrac{\rho}{2\sqrt{\pi\beta}}\right] B \\ \left(p + \rho u^2\right) A^{\pm} \pm \left[\dfrac{\rho u}{2\sqrt{\pi\beta}}\right] B \\ \left(pu + \rho u e\right) A^{\pm} \pm \left[\left(\dfrac{p}{2}\right) + \rho e\right]\left[\dfrac{1}{2\sqrt{\pi\beta}}\right] B \end{bmatrix} \tag{4}$$

KFMG (Kinetic Flux Vector Splitting on Moving Grid) is obtained by considering a 1-D finite volume $a(t) \leq x \leq b(t)$ with moving boundary points [7], that is,

$$\frac{\partial}{\partial t}\int_{a(t)}^{b(t)} f\,dx + (v - w_b)F_b - (v - w_a)F_a = 0 \tag{5}$$

where w_a, w_b are velocities of grid points a, b and F_a, F_b are Maxwellians at these points. Taking moments of (5) gives split fluxes on moving boundaries

$$G^{\pm} = \begin{bmatrix} \rho(u-w) A^{\pm} \pm \left[\dfrac{\rho}{2\sqrt{\pi\beta}}\right] B \\ \left(p + \rho(u-w)^2\right) A^{\pm} \pm \left[\dfrac{\rho(u-w)}{2\sqrt{\pi\beta}}\right] B \\ (u-w)\left(p + \rho e\right) A^{\pm} \pm \left[\left(\dfrac{p}{2}\right) + \rho e\right]\left[\dfrac{1}{2\sqrt{\pi\beta}}\right] B \end{bmatrix} \tag{6}$$

The least squares kinetic upwind method (LSKUM) is obtained by approximating the spatial derivative through use of least squares,

$$F^{(1)}_{x_o} = \left(\frac{\partial F}{\partial x}\right)_0 = \frac{\sum_i \Delta F_i \,\Delta x_i}{\sum_i \Delta x_i^2} \tag{7}$$

where P_o is the point at which $\left(\dfrac{\partial F}{\partial x}\right)_0$ is to be approximated, $P_i \in N(P_0)$, $N(P_o)$ = stencil of nodes or points in the neighborhood of P_o, $\Delta F_i = F_i - F_0$, $\Delta x_i = x_i - x_0$ and $F^{(1)}_{xo}$ is a first order accurate derivative. Second order accuracy is achieved by a defect correction step

$$F^{(2)}_{x0} = \frac{\sum \Delta\widetilde{F}_i \,\Delta x_i}{\sum \Delta x_{i^2}}, \quad \Delta\widetilde{F}_i = \Delta F_i - \frac{\Delta x_i}{2}\left(F^{(1)}_{xi} - F^{(1)}_{xo}\right) \tag{8}$$

Use of $F^{(2)}_{xo}$ instead of (7) leads to the usual second order accurate LSKUM. It is interesting to note that ΔF_i used in first order LSKUM is a difference between nonnegative distributions but $\Delta\widetilde{F}_i$ used in second order LSKUM is not having this property. It is desirable to have a second order accurate LSKUM having the structure of first order accurate LSKUM. Such a consideration leads to q-LSKUM or LSKUM based on entropy variables. The entropy variables (called q-vector) have been introduced by Deshpande [12] for casting Euler equations in symmetric hyperbolic form and they are functions of primitive variables and are defined by

$$q^T = [q_1, q_2, q_3] = \left[\ln\rho + \frac{\ln\beta}{\gamma - 1} - \beta u^2, 2\beta u, -2\beta \right] \quad (9)$$

An interesting characteristic of q variables is that linear interpolation of entropy variables corresponds to taking weighted geometric mean of Maxwellians. The eqn. (8) which can be written as

$$\Delta\widetilde{F}_i = \left(F_i - \frac{\Delta x_i}{2} F_{xi}^{(1)} \right) - \left(F_0 - \frac{\Delta x_i}{2} F_{x_o}^{(1)} \right) \quad (10)$$

is a starting point for q-LSKUM in which (10) is replaced by

$\Delta\widetilde{F}_i = \widetilde{F}_i - \widetilde{F}_0$, $\widetilde{q}_i = q_i - \frac{\Delta x_i}{2} q_{xi}$, $\widetilde{q}_o = q_o - \frac{\Delta x_i}{2} q_{xo}^{(1)}$ and $\widetilde{F}_i$, $\widetilde{F}_o$ are Maxewellians corresponding to $\widetilde{q}_i$ and $\widetilde{q}_o$.

The KSPH [6] is a combination of PVU and LSKUM type discretisation of spatial derivatives. The LSKUM – MG[10] starts with

$$\left(\frac{\partial f}{\partial t} + w \frac{\partial f}{\partial x} \right) + (v - w) \frac{\partial f}{\partial x} = 0 \quad (11)$$

Where w = grid point velocity to be specified separately.

Define

$$\left(\frac{\partial f}{\partial t} + w \frac{\partial f}{\partial x} \right) = \left(\frac{\partial f}{\partial t} \right)_{moving}, \quad \bar{v} = v - w \quad (12), (13)$$

We then have

$$\left(\frac{\partial f}{\partial t} \right)_{moving} + \bar{v} \frac{\partial f}{\partial x} = 0, \quad \frac{dx}{dt} = w \quad (14)$$

And again LSKUM can be used to obtain discrete approximation to f_x leading to LSKUM-MG (LSKUM on moving grids).

III. RESULTS

Several 2-D and 3-D codes have been developed in CFD Centre based on the numerical schemes described above. BHEEMA has been developed jointly by DRDL and IISc for computing inviscid compressible flows around flight vehicle configurations. It has been used by Kulkarni et al [13] for computing shock oscillations ahead of an intake of a ramjet engine. BHEEMA has also been used by Sekar et al[14] for jet plume-deflector interaction of interest to aerodynamics of launcher. Mathur & Deshpande have developed 2D KFVS code for extensively studying cell-centred FVM code on structured meshes, unstructured meshes and cartesian meshes. They have used reconstruction for enhancing spatial accuracy. Krishnamurthy et al [8] have developed KFMG finite volume 2-D code for aerodynamic problems requiring moving grids. This has been used for computing unsteady flow around oscillating airfoils and for performing aeroelastic study with pitch & plunge model coupled with KFMG code. Ghosh & Deshpande[9] have developed 2D LSKUM code for studying flows around airfoil. This has been further developed by Ramesh et al[15] for computing flows on all types of meshes – structured, unstructured, cartesian, chimera meshes. Ramesh et al[15] have developed SUPER BHEEMA code based on 3D LSKUM capable of operating on a variety of meshes – stacked structured mesh, chimera meshes for dealing with deflected control surfaces with deflection, stacked triangular meshes, structured tetrahedral meshes and so on. Dauhoo et al [5] have studied q-LSKUM in more details and both Mathur et al and Dauhoo et al[4,5] have shown that q-KFVS and q-LSKUM are superior to usual cell centered KFVS & LSKUM. Recently Ramanan et al [16] have extended KFVS to viscous flows using flux splitting based on Chapman-Enskog distribution. They have applied KFVS – CE to 2D to shock wave boundary layer problem quite successfully and have captured many fine features of flow. Kulkarni and Deshpande [17] have subjected BHEEMA-PVU and BHEEMA-KFVS to a very extensive Verification and Validation studies. The kinetic numerical schemes have come a long way in the last two decades and have now become a mature tool in the aerodynamic design and analysis problems for practical configurations.

REFERENCES

[1] SM Deshpande, "A second order accurate kinetic theory based method for Inviscid compressible flows". NASA Technical paper 2613, 1986.

[2] JC Mandal and SM Deshpande, "Kinetic Flux Vector Splitting (KFVS) for Euler Equations", Computers and Fluids, vol 23, No. 2, pp. 447-478, 1994.

[3] SV Raghuram Rao and SM Deshpande, "Peculiar Velocity based Upwind Method for Inviscid Compressible Flows", Computational Fluid Dynamics Journal of Japan Society of CFD, vol 3, No. 4, pp. 415-432, Jan 1995.

[4] AK Ghosh, JS Mathur and SM Deshpande, "q-KFVS Scheme - A New Higher Order Kinetic Method for Euler Equations", 16th ICNMFD, Aracachon, France, 1998.

[5] Z M Dauhoo, S M Deshpande, "q-LSKUM-A new higher order kinetic method for Euler equations entropy variables", ISCFD 99, Bremen, Germany, Sept 6-10, 1999

[6] B. Manoj Kumar, S.V. Raghurama Rao and S.M. Deshpande, "Kinetic Smooth Particle Hydrodynamics method using Peculiar Velocity based Upwinding and Least Squares", Fluid Mechanics Report 98 FM 2, Dept. of Aerospace Engineering, Indian Institute of Science, Bangalore

[7] PS Kulkarni and SM Deshpande, "Kinetic Flux Vector Splitting(KFVS) on Moving Grids for Unsteady Aerodynamics", 3rd ECCOMAS, Paris, 1996. Also FM Report No. 95 FM 6, 1995.

[8] R Krishnamurthy, BS Sarma and SM Deshpande, "Kinetic Flux Splitting Scheme for Unsteady Euler Equations on Moving Grids", ACFD3, Bangalore, Dec 1998.

[9] A K Ghosh & SM Deshpande "Least squares kinetic upwind method for Inviscid compressible flows" AIAA 12th CFD Conference, San Diego, California USA, June 19-22, 1995, AIAA-95-1735.

[10] SM Deshpande, PS Kulkarni and AK Ghosh, "New Developments in Kinetic Schemes", Computers Math. Applications, vol 35. No. 1/2, pp 75-93, 1998

[11] V Ramesh, "Least Squares Kinetic Upwind Method on Moving Grids", 99 FM 1, Jan. 1999, Fluid Mechanics Report, Department of Aerospace Engineering, Indian Institute of Science.

[12] SM Deshpande, 'On the Maxwellian Distribution, Symmetric Form and the Entropy conservation for the Euler Equations'. NASA Technical paper 2583, August 1986.

[13] PS Kulkarni, S.Sekar, M Nagarathinam and SM Deshpande, "Numerical Simulation of Pressure Oscillation using BHEEMA for a Generic Intake", ACFD3, Bangalore, Dec 1998, Also JATP Report Dec 1997.

[14] S. Sekar, K Anandhanarayanan, R Krishnamurthy and SM Deshpande, "Numerical Simulation of Rocket Plume-Flame Deflector Interaction", ISCFD 99, Bremen Germany, Sept 6-10, 1999

[15] V. Ramesh, AK Ghosh & SM Deshpande, Sept. 1997 "Computation of three dimensional Inviscid compressible flows using least squares kinetic upwind method (LSKUM)" Fluid Mechanics Report 97 FM 8, Dept. of Aerospace Engineering, Indian Institute of Science, Bangalore.

[16] S Ramanan, B Manoj Kumar, K Anandhanarayanan, SV Raghurama Rao and SM Deshpande "New Boltzmann schemes for Viscous Flows". Asian Conference in CFD (ACFD3), Dec. 7-11,1998, Bangalore.

[17] PS Kulkarni and SM Deshpande, "Verification and Validation of Upwind Euler Solvers for Aerodynamic Design and Analysis of 3D Configurations", 8th ACFM Dec 6-10, 1999, China.